Linear Algebra Common Topics List[1]

1. **Computations and Proofs**
   (a) The course should include both computations and proofs
2. **Geometry and Algebra of Vectors**
   (a) Vector addition and scalar multiplication
   (b) Linear combinations
3. **Systems of Linear Equations**
   (a) Linear versus nonlinear equations
   (b) Homogeneous versus nonhomogeneous systems
   (c) Coefficient matrix and augmented matrix
   (d) Row echelon form and reduced row echelon form
   (e) Gaussian elimination
   (f) Elementary row operations
   (g) Superposition of linear systems: Relationship between the solutions sets of associated homogeneous and nonhomogeneous systems
4. **Matrices**
   (a) Matrix addition and multiplication
   (b) Properties of matrix multiplication
   (c) Transposes and symmetric matrices
   (d) Matrix Inverses and Invertibility
   (e) Determinant
      - Computing determinants
      - Effect of elementary row operations
      - Relationship to invertibility
   (f) Trace
5. **Vector Spaces**
   (a) Definition of a vector space
   (b) Examples including $\mathbb{R}^n$ and function spaces
   (c) Subspaces, including lines and planes through the origin as examples
6. **Bases**
   (a) Span of a set of vectors

[1]This list was approved by the department on 5/3/19.

(b) Linear dependence and independence

(c) Basis for a vector space

(d) Dimension of a vector space

7. **Linear Transformations**

   (a) Definition of a linear transformation

   (b) Relationship between matrices and linear transformations

   (c) Row space, column space, and null space of a matrix

   (d) Rank and nullity of a matrix; rank-nullity theorem

   (e) Image and kernel of a linear transformation

   (f) The columnspace of a matrix is the image of the transformation

8. **Eigenvectors and Eigenvalues**

   (a) Formal definitions of eigenvectors and eigenvalues

   (b) Characteristic polynomial and characteristic equation

   (c) Properties of eigenvectors, including relationships to the transpose and inverse

   (d) The determinant is the product of the eigenvalues

9. **Similarity and Diagonalization**

   (a) Similarity invariants, like eigenvalues, determinant, and trace

   (b) Definition of diagonalizable matrix

   (c) If a matrix has distinct eigenvalues it is diagonalizable

10. **Dot Products and Inner Products**

    (a) Definitions

    (b) Vector norms and angles

    (c) Orthogonal Vectors

11. **Projection**

    (a) Projection onto a vector

    (b) Projection onto a subspace

    (c) Orthogonal complement of a subspace

    (d) Orthogonal decomposition of a vector

    (e) The nullspace and rowspace of a matrix are orthogonal complements

12. **Orthogonal bases**

    (a) An orthogonal set is linearly independent

    (b) Orthonormal basis

Examples of Additional Topics

- Complex vector spaces
- Gram-Schmidt process
- Orthogonal Matrices
- LU factorization
- QR factorization
- Markov Chains
- Principal Component Analysis
- Least Squares Approximation
- Distance between points, lines, and planes
- Matrix Block Multiplication
- Cramer's Rule
- Iterative Methods
- Singular Value Decomposition
- Cayley-Hamilton Theorem
- Jordan Canonical Form and Rational Canonical Form
- Linear Differential Equations as the kernel of a linear transformation