Cinematic Box-Office Dynamics: An Overview of a Particular Application of Ordinary Differential Equations to the Time Evolution of Theatrical Film Grosses

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Cinematic Box-Office Dynamics is the name given to the study of the way in which theatrically released films earn money over time. This phenomenon is an active area of research in economics and mathematics. We will present an overview of several problems involved in creating effective mathematical descriptions of the evolution over time of a film's earning potential at the box-office. We will also discuss some attempts at developing and testing mathematical models involving ordinary differential equations that can simulate or approximate the box-office dynamics of certain classes of films.

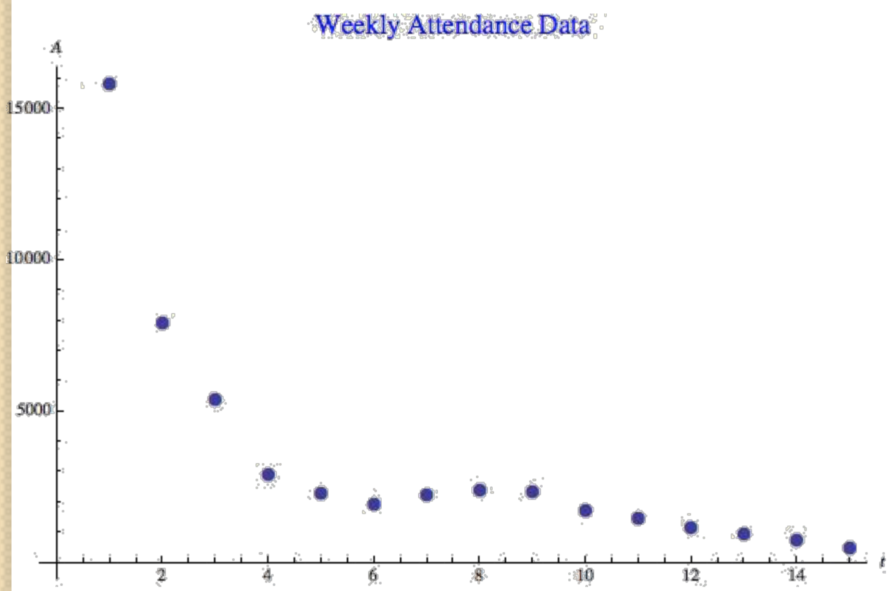
Outline

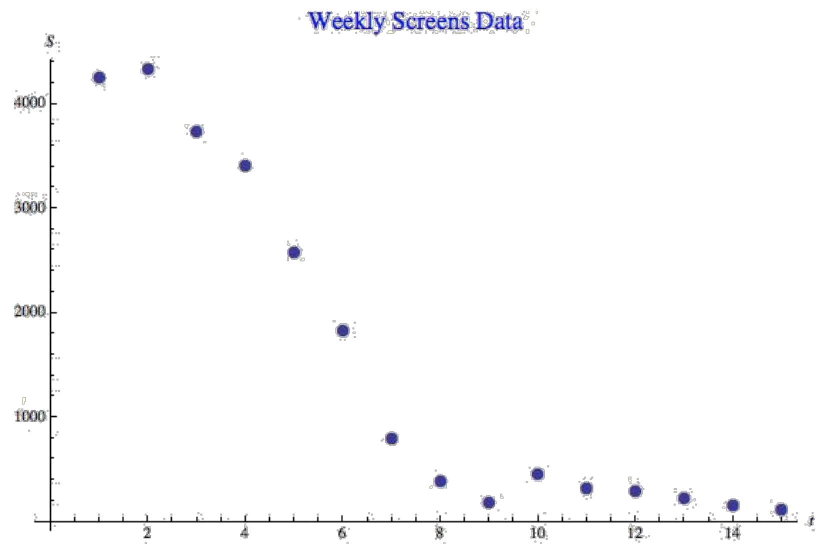
- Introduction to Cinematic Box-Office Dynamics
  - Assumptions, Variables, Observations, Concepts
  - Graphs of typical movie data (2007-2015)
- Edwards-Buckmire Model (EBM)
  - Summary of Parameters
  - Dimensionless version
  - Typical solution curves
- Modified EBM
- Numerical Results
- Ongoing Work
  - The Holy Grail: *A Priori* Prognostication of Final Gross
  - Sequels: Relationships Between Related Films
- Conclusions

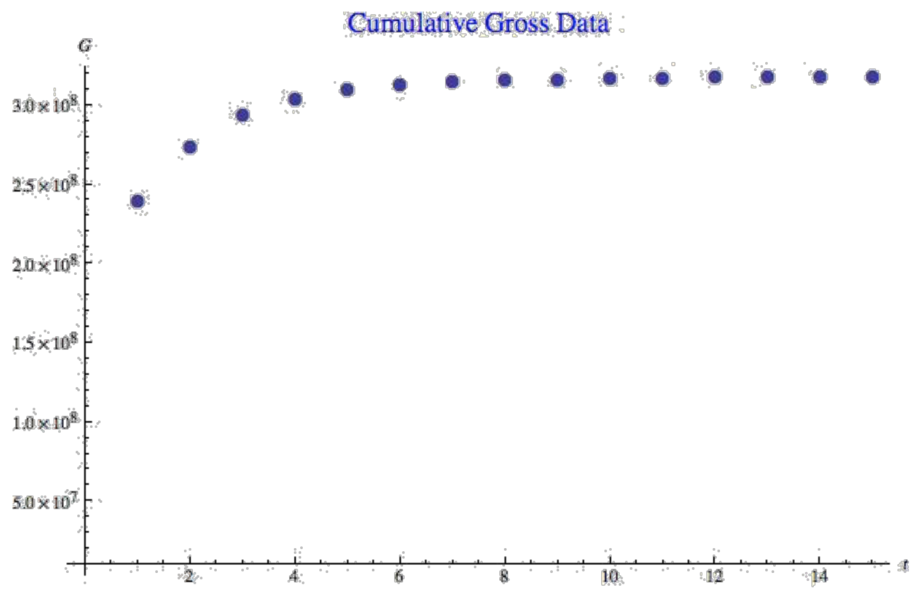
Introduction to Cinematic Box-Office Dynamics

- **Assumptions**
  - Movies are released on a number of screens in North America that changes weekly
  - There is available (daily and weekly) data on how much money the film makes
  - Variables are continuous functions of time
  - Deterministic model, not Stochastic
  - Ordinary differential Equation
- **Variables**
  - $G(t)$: cumulative gross receipts of a movie M
  - $S(t)$: number of screens on which M is exhibited
  - $A(t)$: normalized weekly revenue of M ($ per screen average)
  - t : time in number of weeks

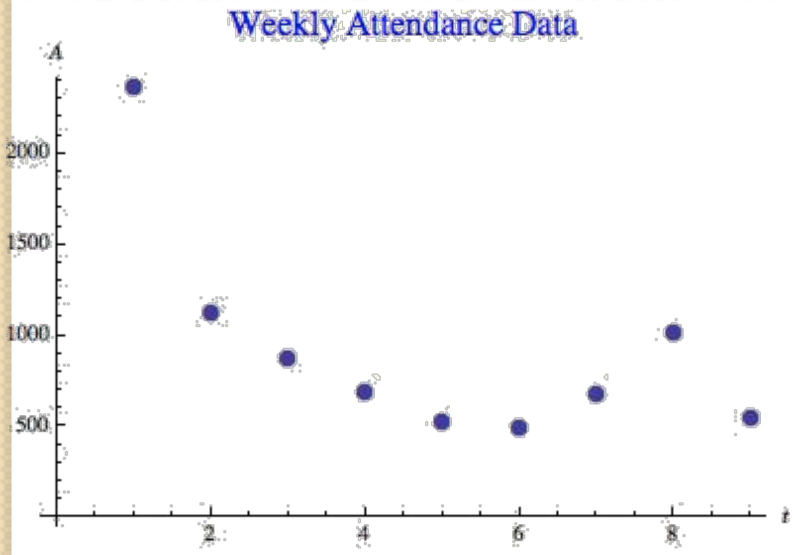
Actual Movie Data: *Spider-Man 3 (2007)*

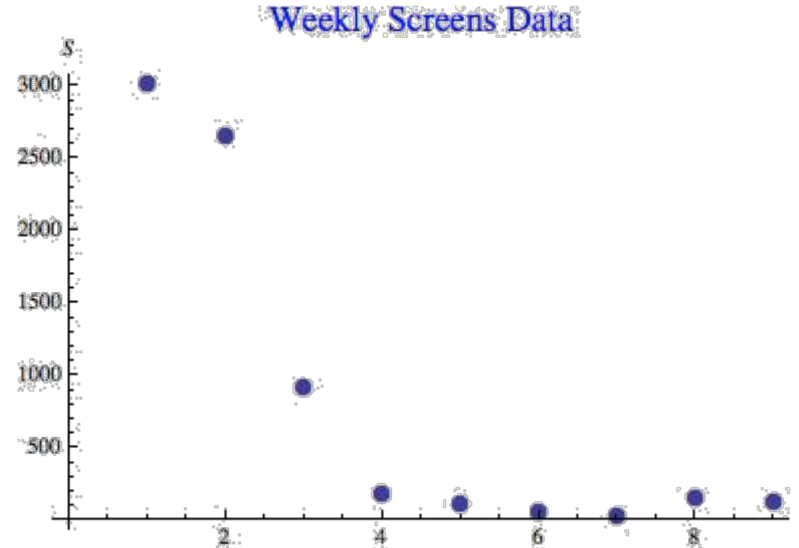


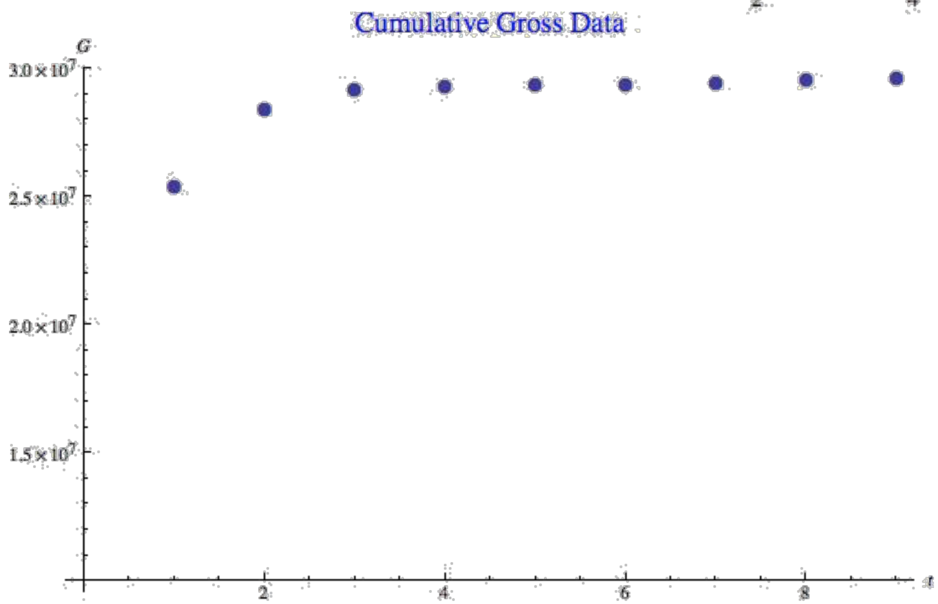




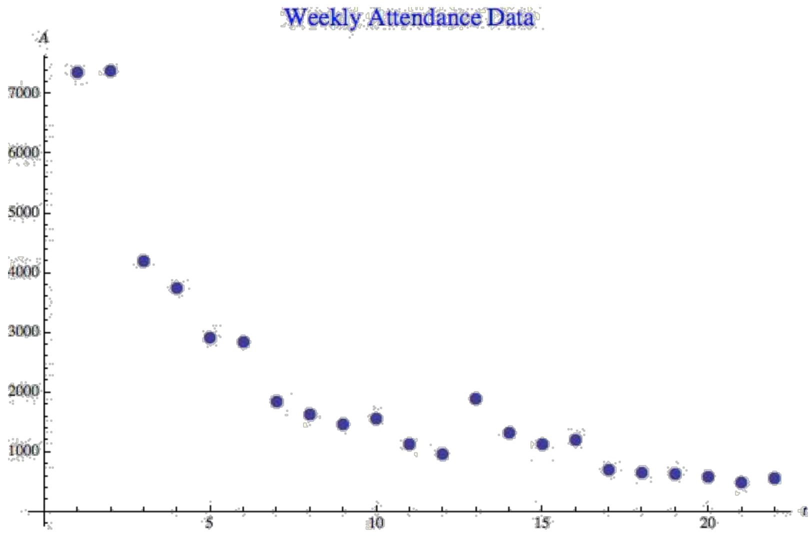
Actual Movie Data: *The Love Guru (2008)*

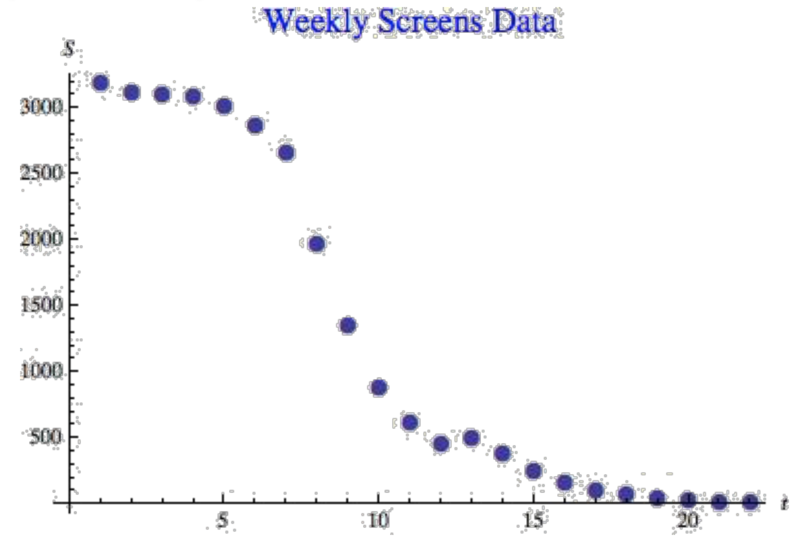


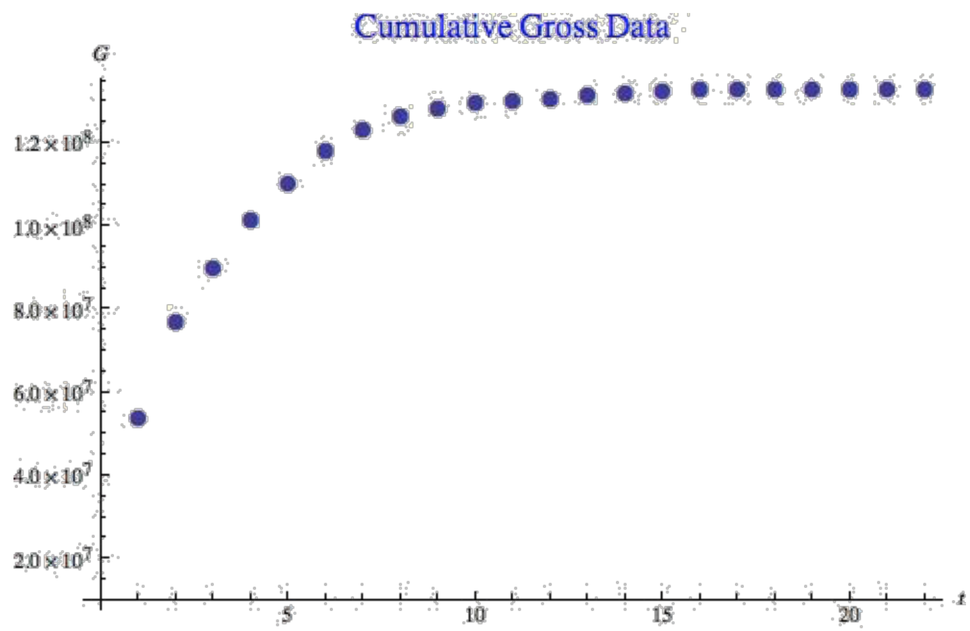




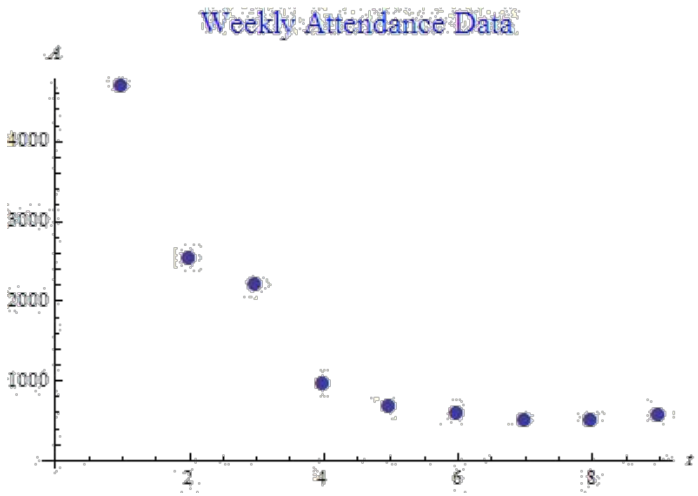
Actual Movie Data: *Taken (2009)*

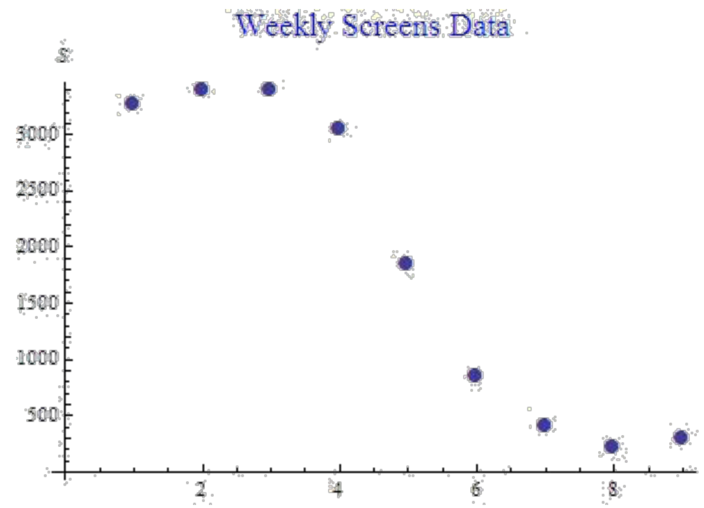


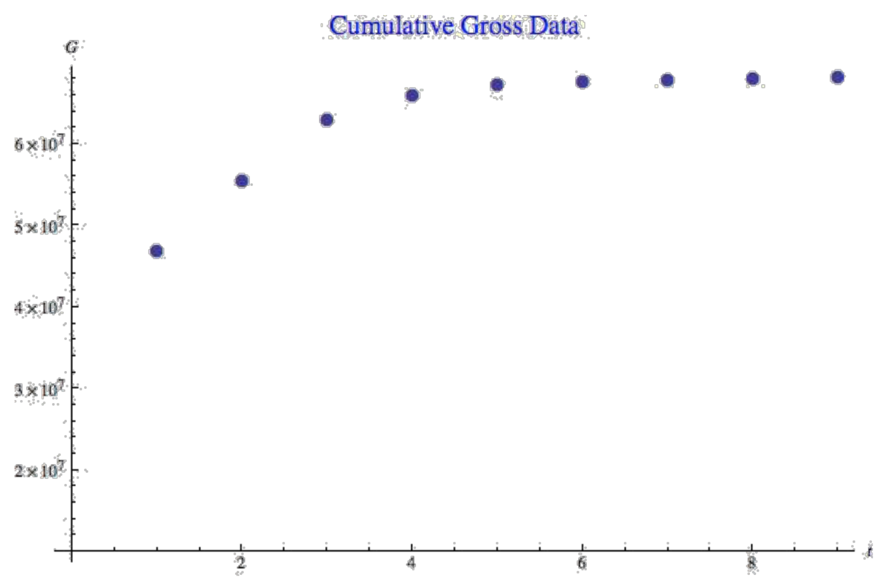




Actual Movie Data: *The Expendables (2010)*







Introduction to Cinematic Box-Office Dynamics

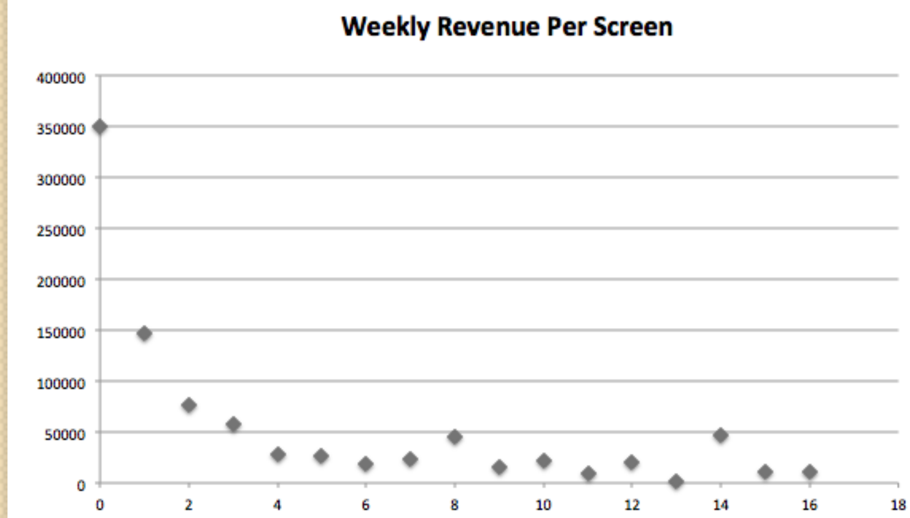
- **Observations**
  - G, A and S have **quasi-exponential** profiles
  - A and S data is much less smooth than G data
  - Blockbuster movies make the bulk of their final gross G_∞ in a brief fraction of their lifespan
  - There is often a contract period where S is almost constant
- **Concepts**
  - Fundamental Law of Cinematic Box-Office Dynamics

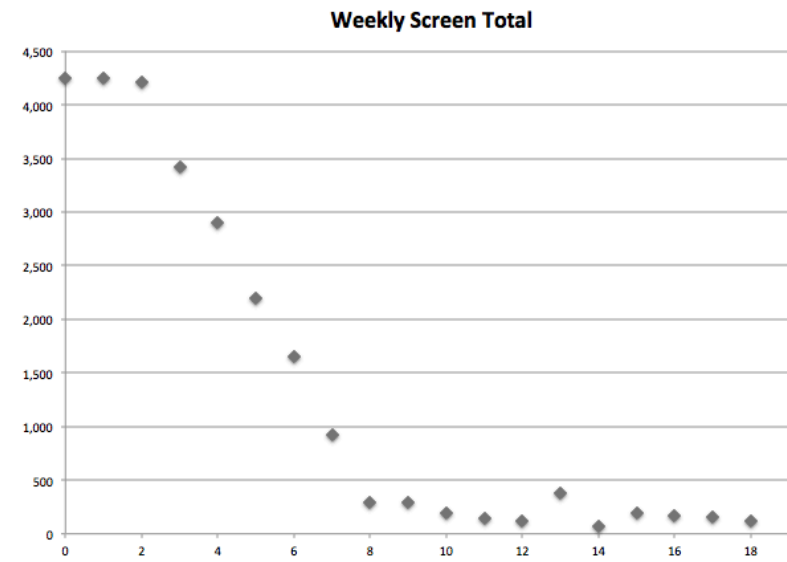
$$G'(t) = S(t)A(t)$$

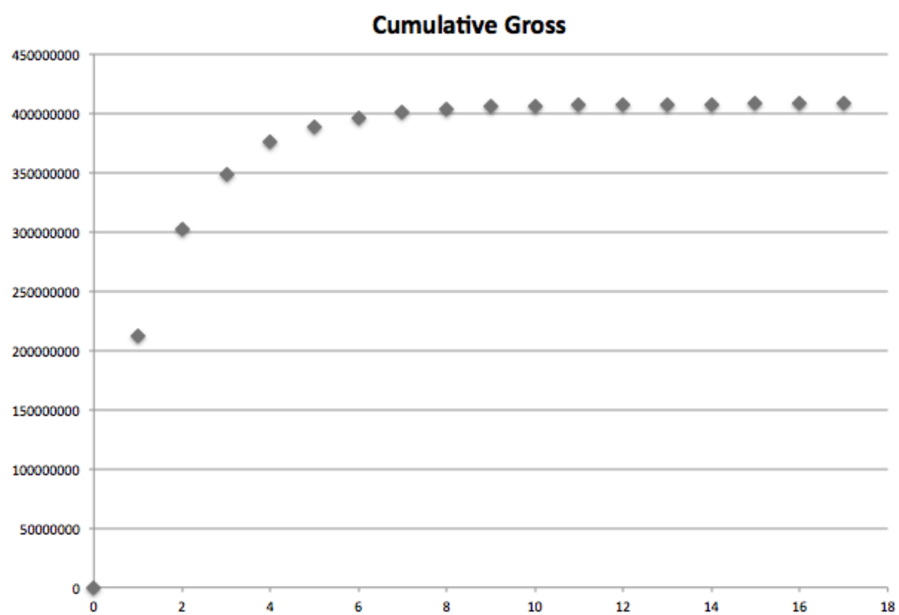
  - The cumulative final gross G_∞

$$G_\infty = \int_0^\infty G'(t)\ dt = \int_0^\infty S(t)A(t)\ dt$$

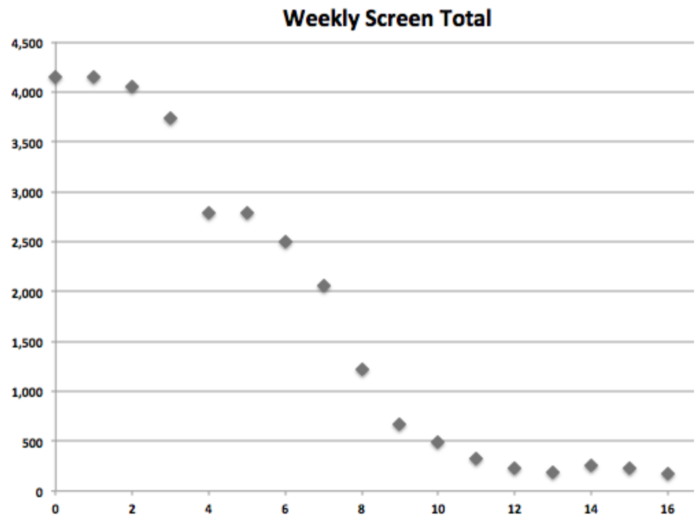
Actual Movie Data: *Iron Man 3 (2013)*

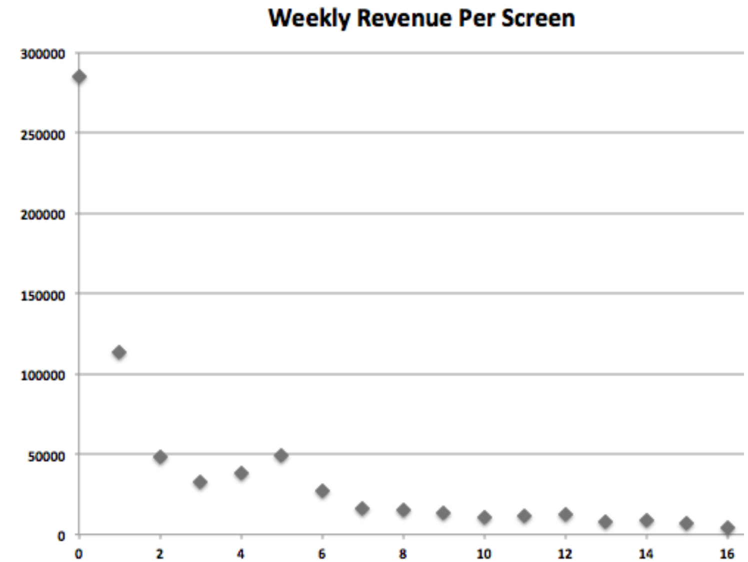


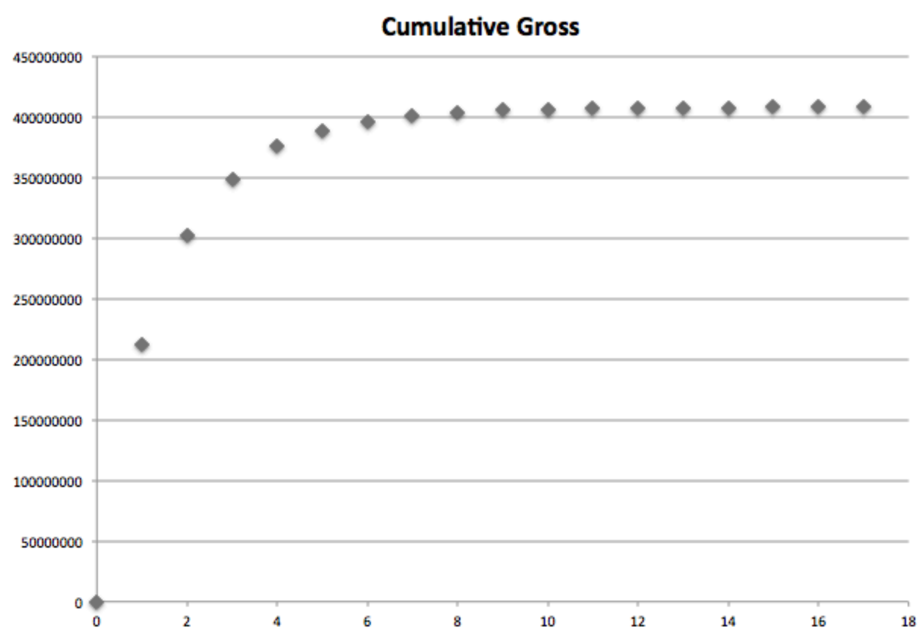




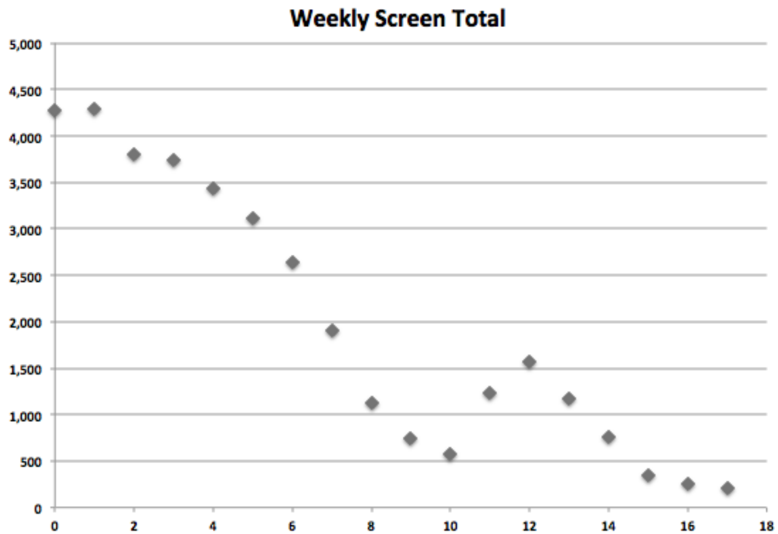
Actual Movie Data: *Mockingjay, Part 1 (2014)*

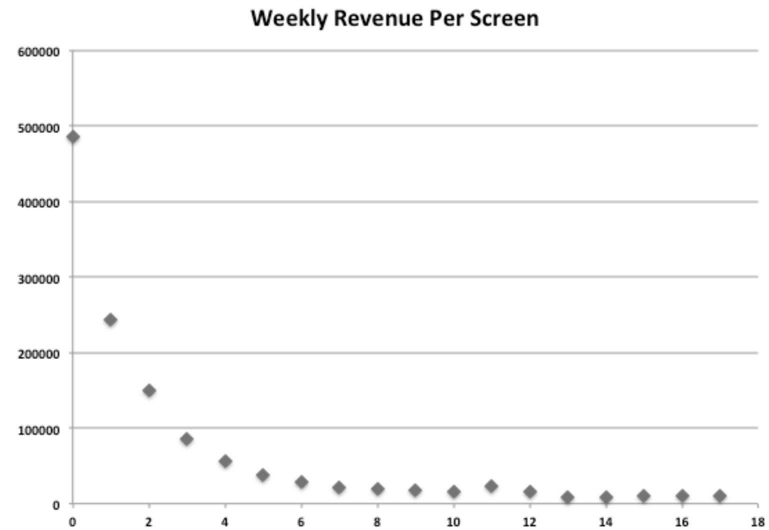


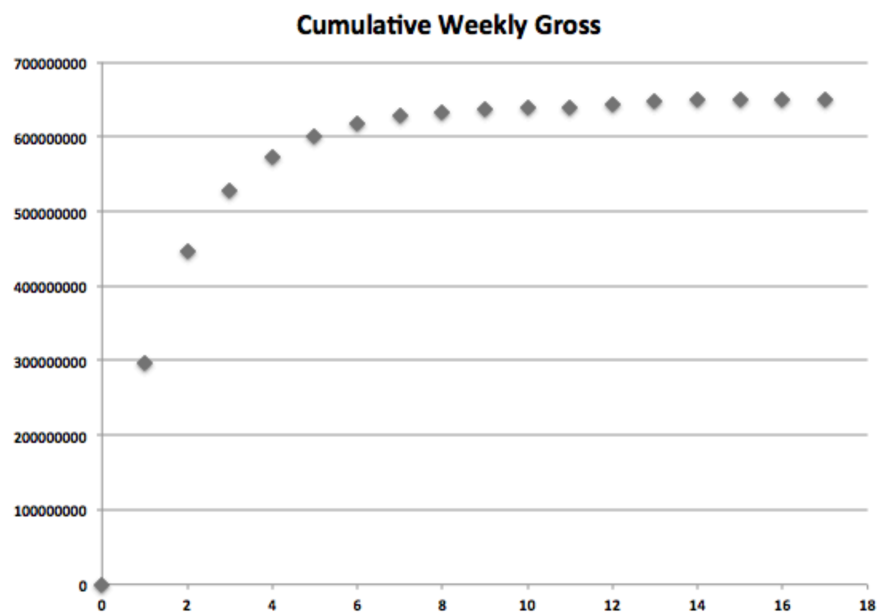




Actual Movie Data: *Jurassic World (2015)*







The (original) Edwards-Buckmire model (2000)

A 3-D system of nonlinear, coupled, autonomous ordinary differential equations with many parameters

$$\frac{dG}{dt} = SA \qquad G(0) = 0$$

$$\frac{dS}{dt} = -\alpha_S \left(S - S_* \frac{A}{A_{\max}} \right) \qquad S(0) = S_0$$

$$\frac{dA}{dt} = -\frac{\alpha_A}{D(1+\varepsilon)} \left[\frac{S}{S+\gamma M} + \frac{\beta}{P} GH_{\%}^2 \right] A \qquad A(0) = A_0$$

EBM Parameters

α_A	:	natural exponential decay rate of A
α_S	:	natural exponential decay rate of S
D	:	average number of repeat viewings
ϵ	:	effect of critical reviews on A
γ	:	effectiveness of marketing budget on A
β	:	effectiveness of word-of-mouth effects on A
P	:	price of a movie ticket
$H_{\%}$	:	percent of people who dislike or hate the film
M	:	marketing budget
A_{max}	:	maximum possible value of A for a given week.
S^*	:	critical threshold between platform and wide release patterns

Dimensionless Version of EBM

A 3-D system of nonlinear, coupled, autonomous ordinary differential equations with 3 parameters

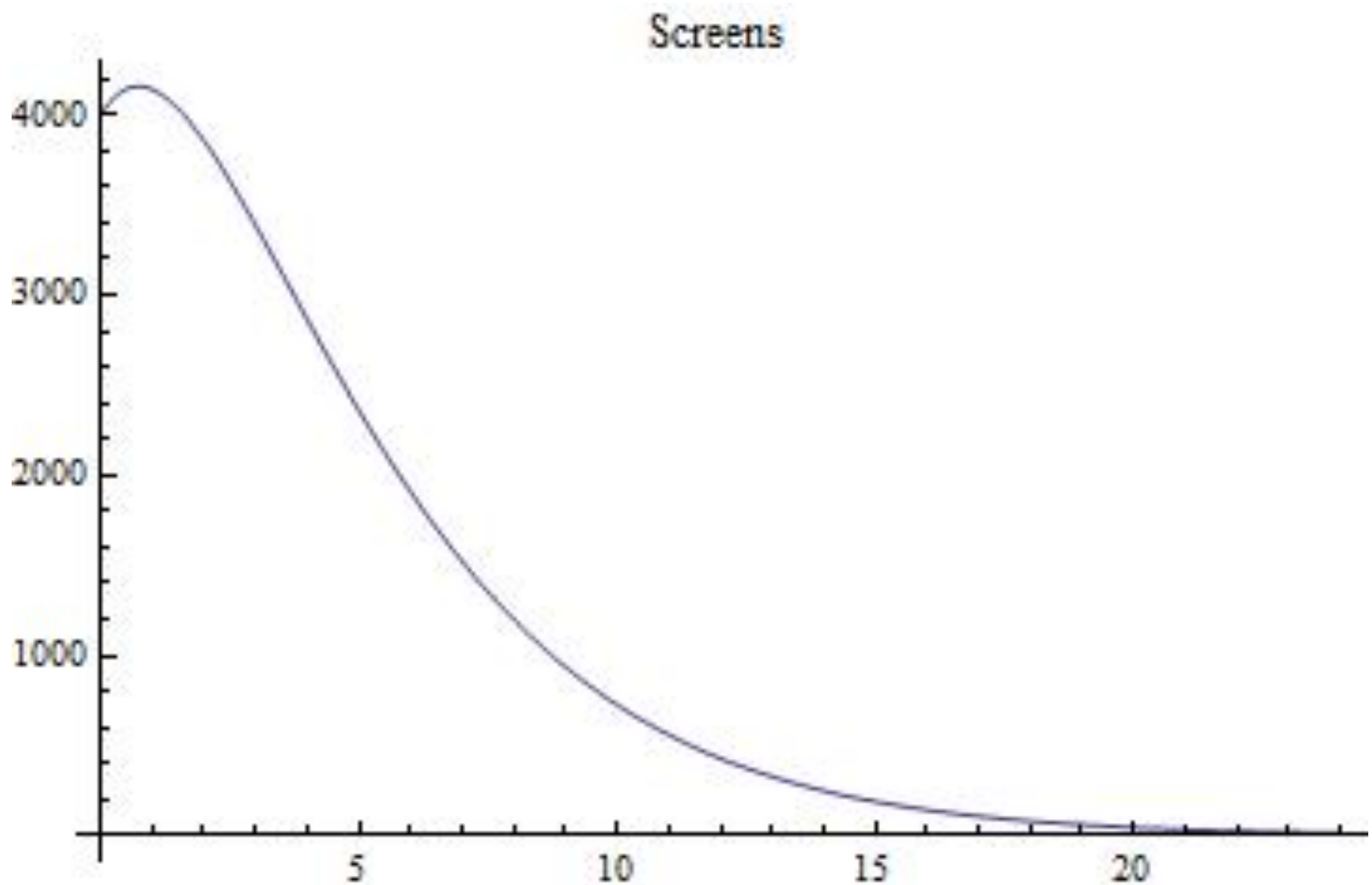
$$\frac{dG}{dt} = SA \qquad \frac{dS}{dt} = -(S - A)$$

$$\frac{dA}{dt} = -\tilde{\alpha}\left[\frac{S}{S+\tilde{\gamma}} + \tilde{\beta}G\right]A$$

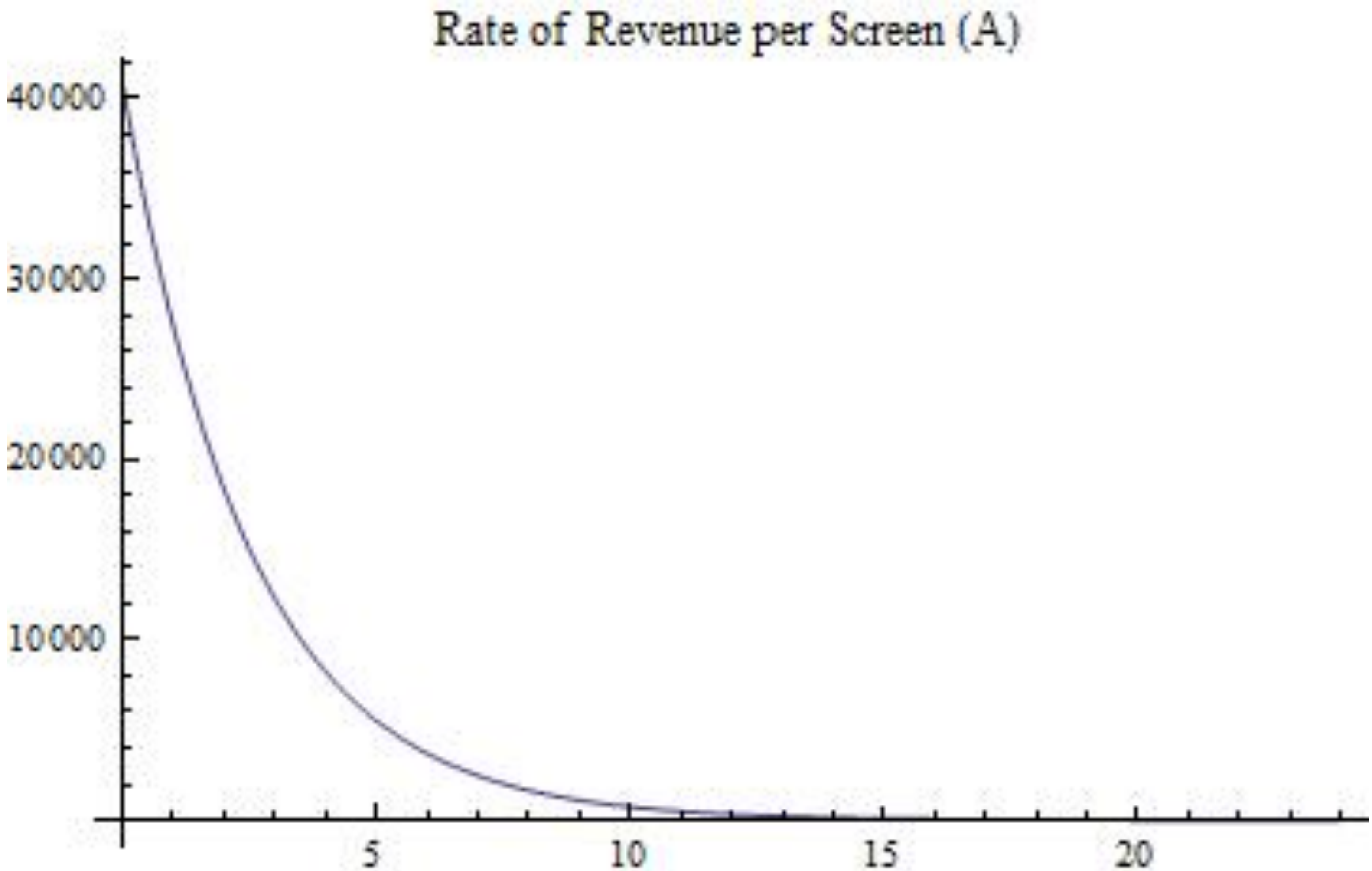
where

$$\tilde{\gamma} = \frac{\gamma M}{S_*} \qquad \tilde{\alpha} = \frac{\alpha_A}{\alpha_S}\frac{1}{D(1+\varepsilon)} \qquad \tilde{\beta} = \frac{\beta H_{\%}^2 S_* A_{\max}}{P\alpha_S}$$

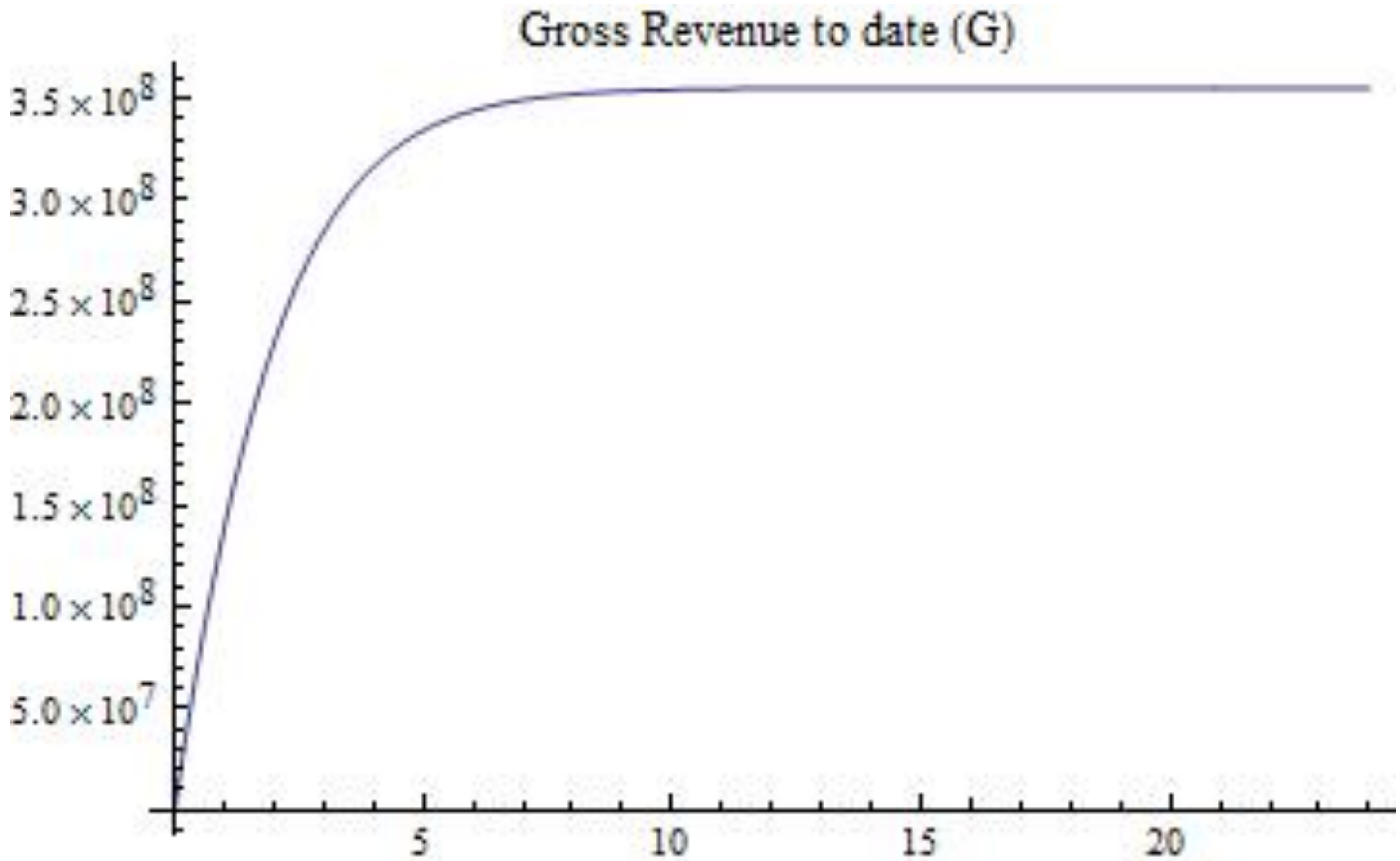
Typical EBM solution curves: *S(t)*



Typical EBM solution curves: *A(t)*



Typical EBM solution curves: *G(t)*



Drawbacks of EBM

- $H_{\%}$ varies with time
- Parameter $(D, \epsilon, \gamma, \beta)$ estimates are difficult to make and somewhat arbitrary
- Most movies have a contract period in which screens is constant, i.e. $S'=0$
- S and A actual data more erratic (i.e. require more complicated equations) than first thought; G is relatively smooth

Modifying the EBM (Edwards, Buckmire & Ortega-Gingrich, 2013)

- Uses an Economics-inspired "demand" model
- Based on quasi-exponential decay form of A and S
- Incorporates fixed contract periods when screens are constant
- Introduces S_{max} (max possible) and S^*(saturation)
- Greatly modifies the A equation
- Both EBM versions have 3 unknown parameters
- Incorporates geographic effects through introduction of μ function that depends on screens

Modified EBM: A Geographic Model of Cinematic Box-Office Dynamics

The function *μ(S)* should satisfy the following conditions

$$s \to 1, \qquad \mu \to a$$
$$s \to \infty, \qquad \mu \to 1$$
$$s = 0, \qquad \mu = 0$$

One selected form of *μ(S)* used is given below (*a=1/T*),
T is total number of movie screens in North America (~40,000)

$$\mu(S) = 1 - (1-a)^S$$

Another form of *μ(S)* is

$$\mu(S) = \begin{cases} \left(\frac{S}{S_*}\right)^\beta, & S \leqslant S_*, \quad S_* = \frac{T^{1/\beta}}{S_{\max}} \\ 1, & S \geqslant S_*. \end{cases}$$

Modified EBM

Another 3-D system of nonlinear, coupled ordinary differential equations

$$G'(t) = S(t)A(t)$$

$$S'(t) = \begin{cases} 0 & \text{if } t \le t_c \\ \alpha_S(A - \kappa S) & \text{if } t > t_c \end{cases}$$

$$A'(t) = \left[-\alpha_A - \frac{S'(t)}{S} + \left(\frac{S'(t)}{\frac{\mu(S)}{\mu'(S)}}\right)\right] A$$

Comparing Original EBM and Modified EBM

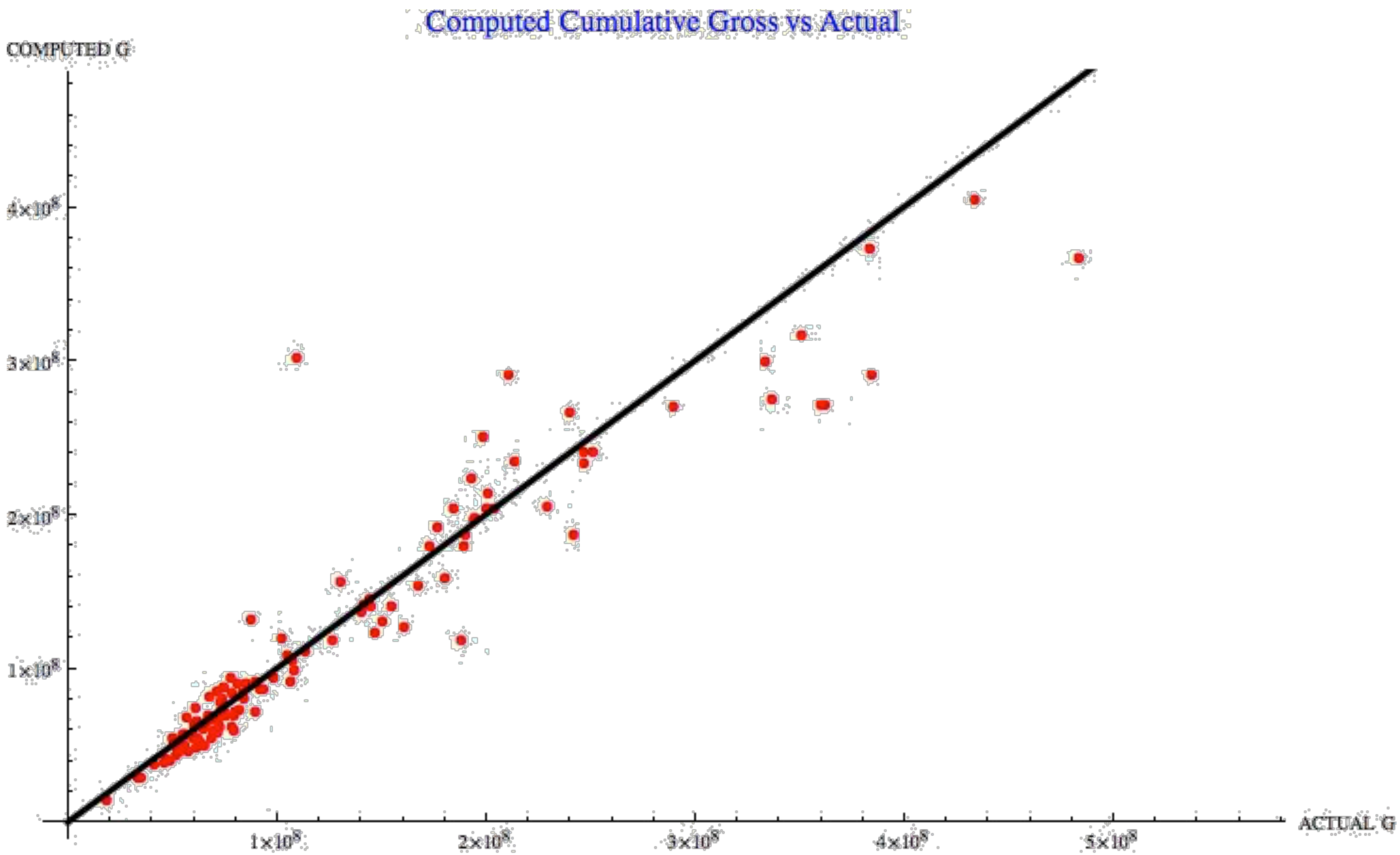
Original EBM	Modified EBM
$\frac{dG}{dt} = SA$	$G'(t) = S(t)A(t)$
$\frac{dS}{dt} = -(S - A)$	$S'(t) = \begin{cases} 0 & \text{if } t \leq t_c \\ \alpha_S(A - \kappa S) & \text{if } t > t_c \end{cases}$
$\frac{dA}{dt} = -\tilde{\alpha}\left[\frac{S}{S + \tilde{\gamma}} + \tilde{\beta}G\right]A$	$A'(t) = \left[-\alpha_A - \frac{S'(t)}{S} + \left(\frac{S'(t)}{\frac{\mu(S)}{\mu'(S)}}\right)\right]A$

Numerical Calculations

- Analyzed119 movies from 2005-2010 (minimum final gross \$50m)
- All dollars were adjusted for inflation to 2005
- Used *Mathematica* to generate numerical solutions to the modified EBM
- Attempted to find "global" values of parameters α_A, α_S and κ that would minimize std. dev. in difference between computed G_∞ and actual G_∞ while minimizing error
- The hope was to find global parameters that could be used in a model to describe cinematic box-office dynamics for all movies

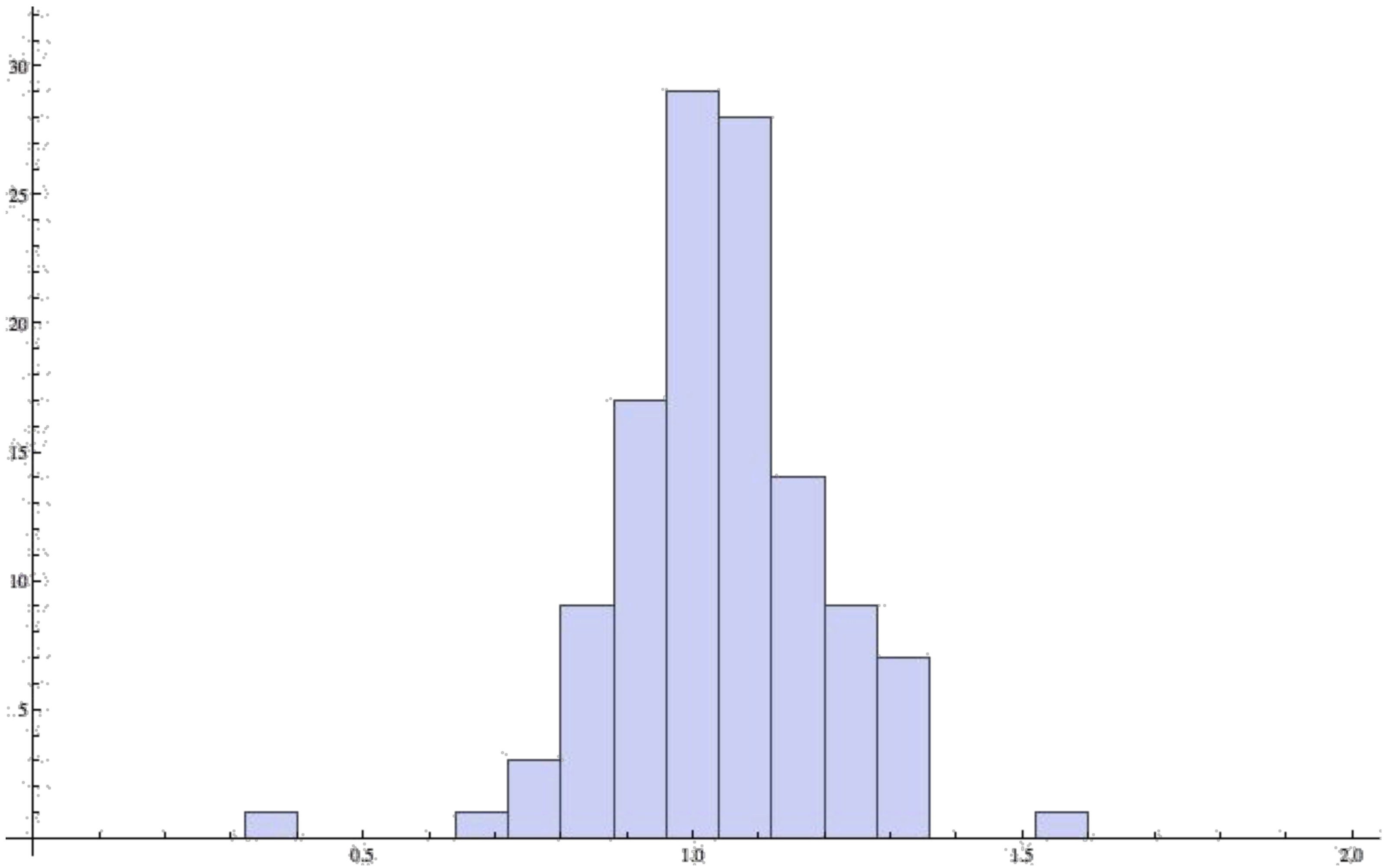
Numerical Results (N=119)

$$\alpha_A = 0.40, \quad \alpha_S = .10, \quad \kappa = .75$$



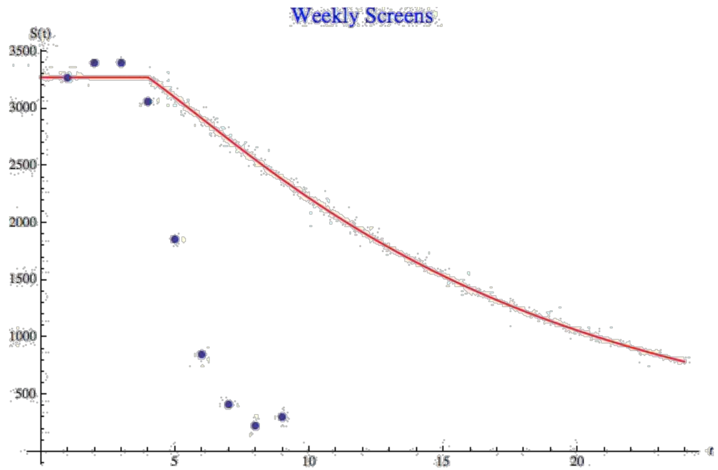
Numerical Results (N=119)

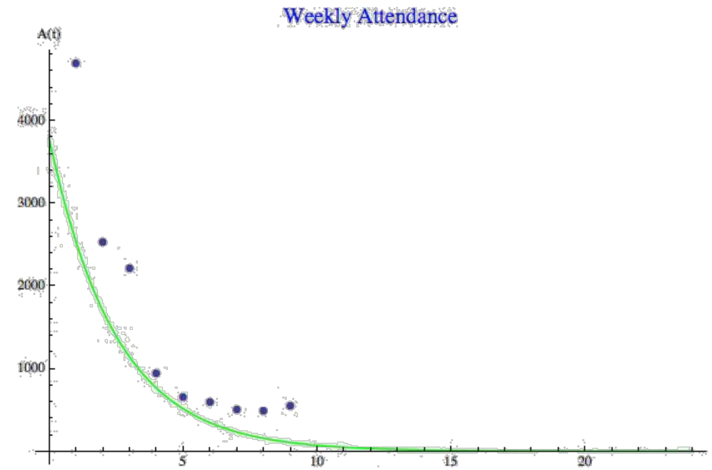
Distribution of G Computed/G Actual as Histogram
mean=1.0389, std. dev.=0.158

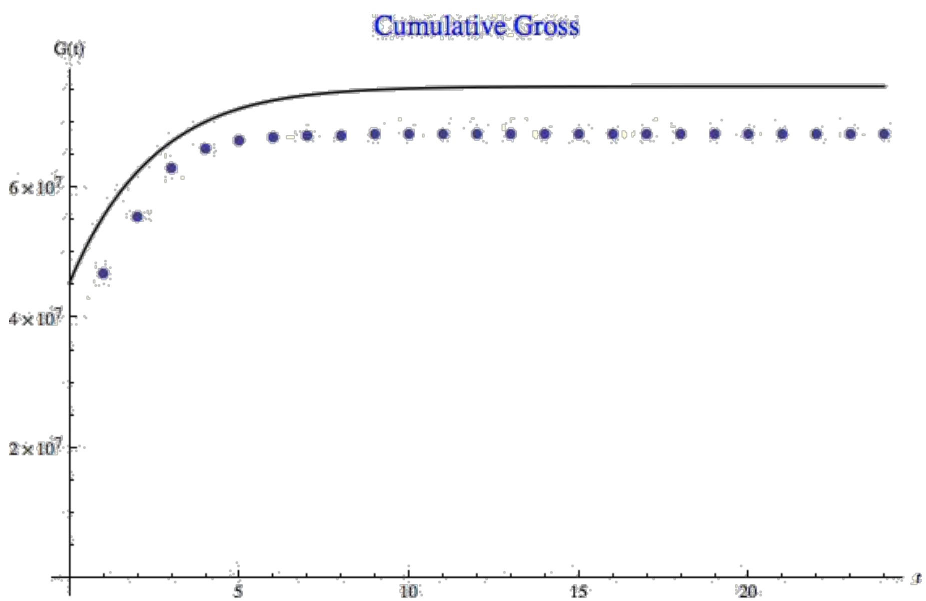


Numerical Results: Using Global Parameters

The Expendables (2010) $\alpha_A = 0.40, \quad \alpha_S = .10, \quad \kappa = .75$





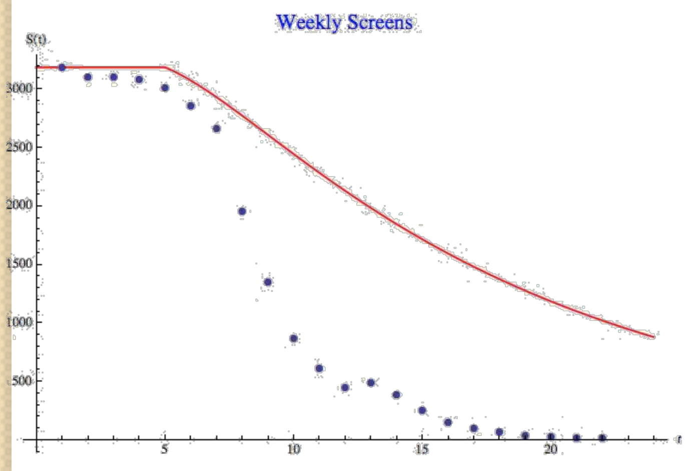


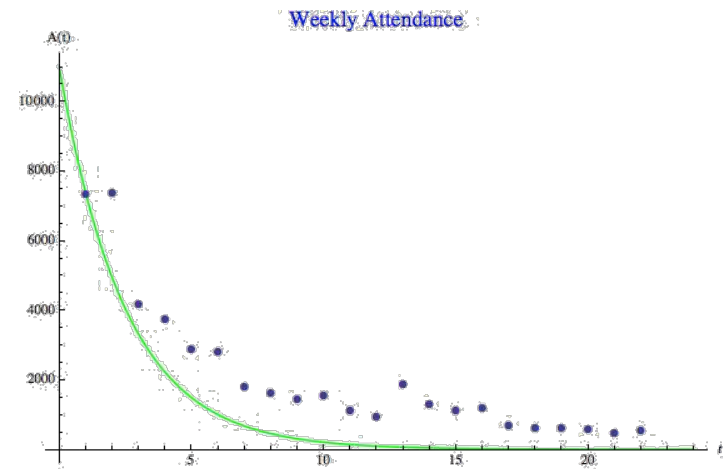
$$\frac{G_\infty^{COMPUTED}}{G_\infty^{DATA}} = 1.1106$$

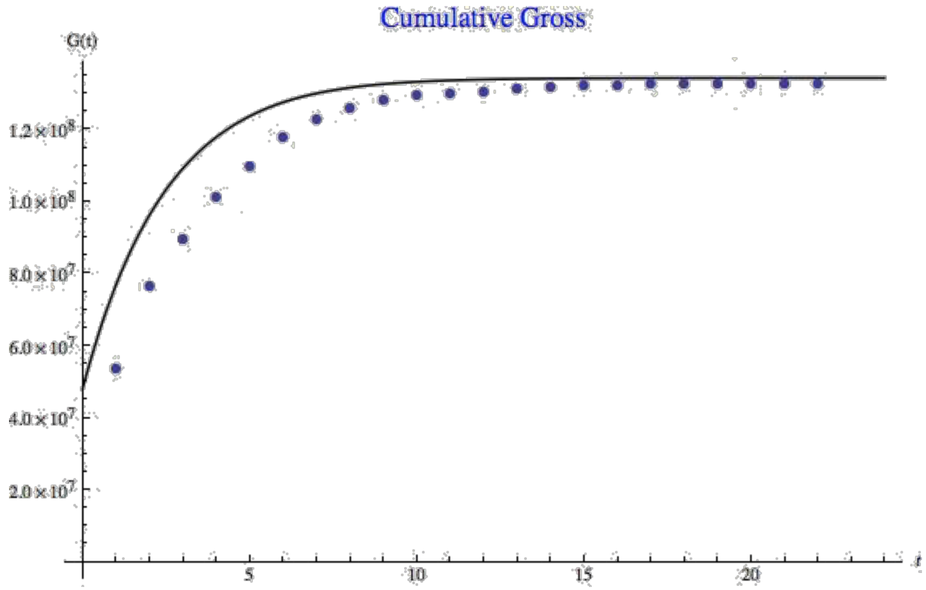
Numerical Results: Using Global Parameters

Taken (2009)

$$\alpha_A = 0.40, \quad \alpha_S = .10, \quad \kappa = .75$$



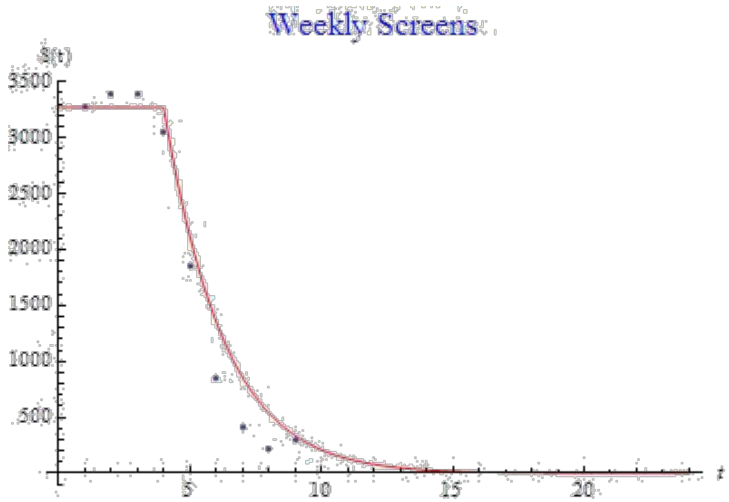


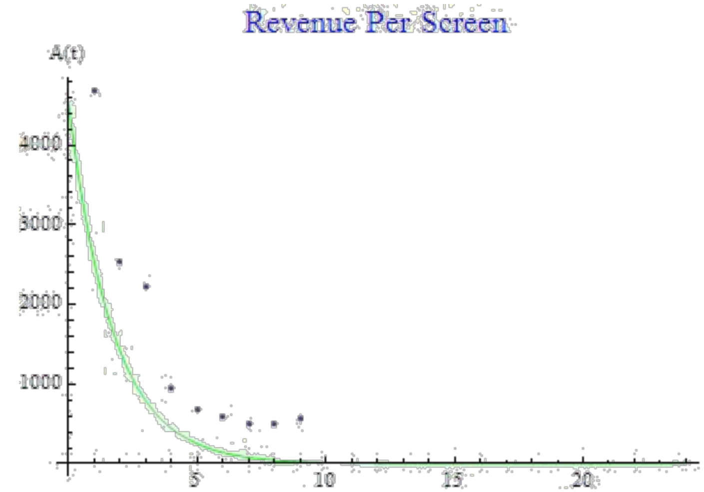


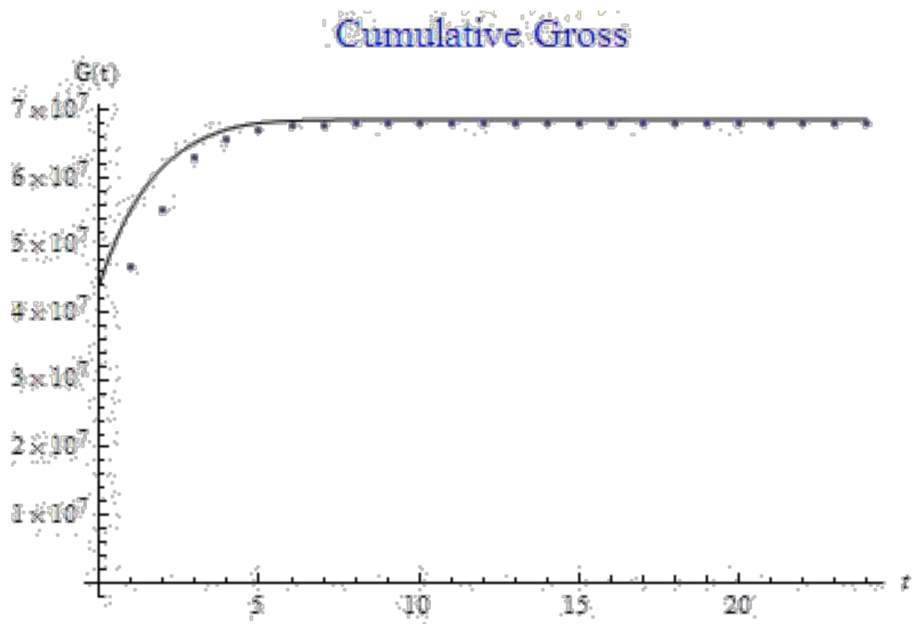
$$\frac{G_\infty^{COMPUTED}}{G_\infty^{DATA}} = 1.01105$$

Numerical Results: Using Chosen Parameters (***The Expendables***)

$$\alpha_A = 0.58, \quad \alpha_S = .70, \quad \kappa = .75, \quad a = 1/40,000$$







$$\frac{G_\infty^{COMPUTED}}{G_\infty^{DATA}} = 1.00794$$

Numerical Results: Conclusions

- The effort to find global values of the parameters that would work for all films was not successful
- For any film it appears as if there exist a set of parameters which can minimize difference between predicted G_∞ and actual G_∞

Ongoing Work

- **"The Holy Grail"** : Predict the final cumulative gross before the movie is released (or very soon after release)
- NO PROGRESS!
- **The sequel problem**: Predict the gross of a sequel based on characteristics of the parent film
- LIMITED PROGRESS!

The *a priori* Problem:

Questions to consider

- How far in advance of a film's release date is it even feasible to try and predict a final gross total?
- How sensitive/dependent is final gross to pre-release audience effects (marketing and advertising, critical reaction, screen release pattern, social media activity)?
- Can we try and predict G_0, and use that to predict G_∞?

The Sequel Problem

- Considered a subset of the *a priori* prediction problem with (possibly) more known information
- How does one define a sequel and differentiate it from a *remake*, *reboot* or *recast*?
- One assumption is that a sequel's first weekend revenue, G_0, may depend on some aspect of the parent
- More likely that G_0 and A_0 depend on awareness of film (which probably depends on marketing budget, *MB*)
- Many more sequels are being released by Hollywood, increasing the importance of the Sequel problem
- Work has been done on 39 parent-sequel pairs between 2005-2012 with limited results

Conclusions

- Accurately predicting the final accumulated gross of any given movie prior to its release is a (very) hard problem
- There exist parameters which can accurately describe the cinematic box-office dynamics of individual films
- There do not seem to exist global parameters for the modified EBM that can describe *all* films
- There is evidence to indicate a relationship between *decay rates* of sequels and parent films
- There is no evidence of relationship between final gross of sequels and parent films
- Access to more individualized movie data (especially proprietary audience awareness tracking polls) would enhance the model

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Questions?

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