Numerical Analysis

Math 370 Fall 1998 — MWF 11:30am - 12:25pm

 — Fowler 127

Class 14: **Monday October 05**

SUMMARY Analyzing Root Finding Algorithms Continued: Newton-Raphson and Secant

READING Burden & Faires, 65–75

The Newton-Rapshon Algorithm

Bisection and False Position are both **globally convergent** algorithms, because, given a bracket which contains a solution, they both will find the solution, eventually.

Newton's Method (and the Secant Method) is very different from these methods, in that instead of needing a bracket where the solution exists [i.e. two function values whose product is negative] one needs a **single** guess of the solution, which has to be "close" to the exact answer, in order for these **locally convergent** to get the solution.

A Derivation of Newton's Method

Write down the first 3 terms of a Taylor expansion of $f(x)$ about the point $(p_0, f(p_0))$

Evaluate this function at the root, the point $(p_1, 0)$ and solve for p_1

This is the iterative step for Newton's Method

Pseudocode for Newton's Method

Example

Consider the function $f(x) = x^2 - A$, where $A > 0$

Compute the Newton iterative step using the above function $f(x)$

Simplify it, so that it look like $x_{n+1} = \dfrac{x_n + A/x_n}{2}$. Recognize this iteration?

Secant Method

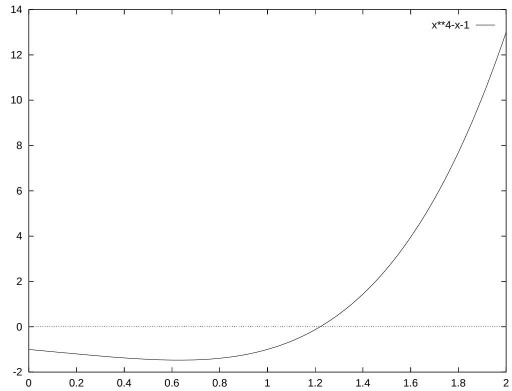
The secant method is very similar to Newton's method, except that instead of actually computing the derivative, one approximates it using a difference quotient. This ends up in making the iterative step look algebraically identical to the one for the False Position method.

Exercise

We will write down the **Secant Method iterative step** below

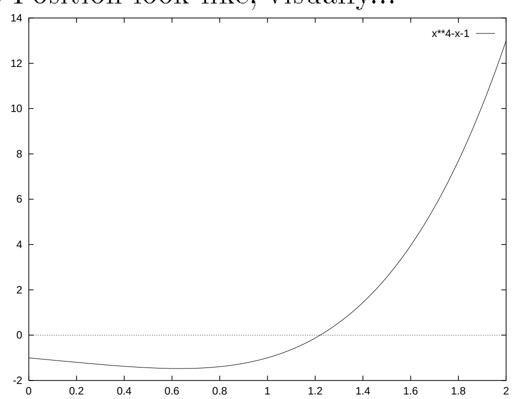
If the iterative step is identical to False Position, how come the Secant Method is not just called the False Position method? Look at the picture...

Secant Method, visually



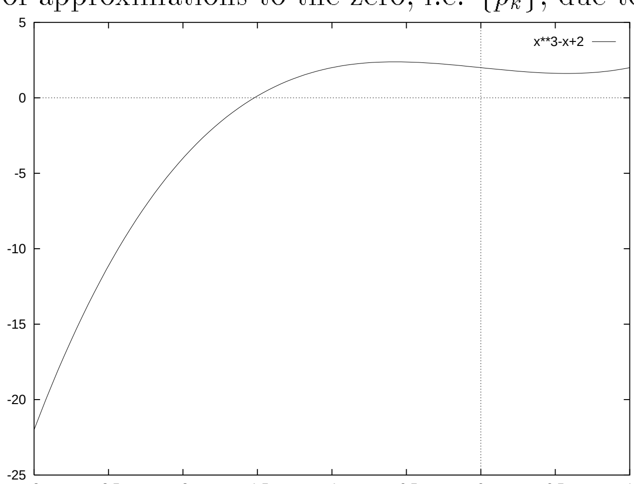
False Position Method, visually

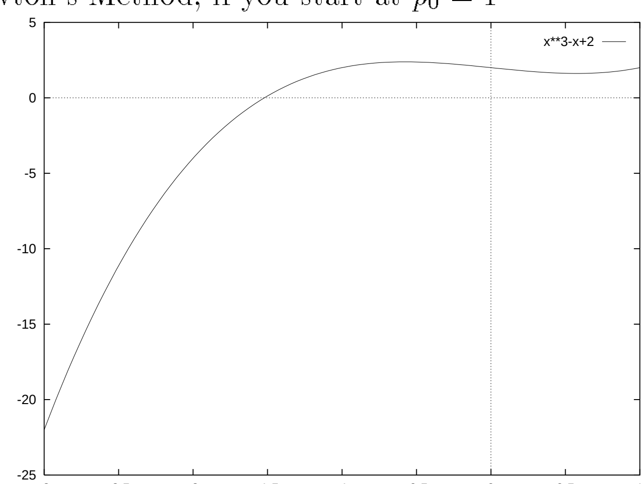
Now, let's recall what False Position look like, visually...



Example

Consider the following function $f(x) = x^3 - x + 2$. On the left figure, draw on the graph the set of approximations to the zero, i.e. $\{p_k\}$, due to Newton's Method, if you start at $p_0 = 1$





On the right figure, draw on the graph the set of approximations produced by Bisection, Secant and False Position (use differently colored pens).