Numerical Analysis

Math 370 Fall 1998 MWF 11:30am - 12:25pm
 Fowler 127

Class 7: **Friday September 18**

SUMMARY Rates of Convergence of Functions and Sequnences
CURRENT READING Burden & Faires (6th edition,) pages 36-37

Rates of Convergence of Functions
If we know that $\lim_{h\to 0} F(h) = L$ and and $\lim_{h\to 0} G(h) = 0$ If a positive constant K exists with

$$|F(h) - L| \leq KG(h), \qquad \text{for sufficiently small } h$$

then we write $F(h) = L + O(G(h))$

This can also be computed using the idea that $F(h) = L + O(G(h))$ if

$$\lim_{h\to 0} \frac{|F(h) - L|}{|G(h)|} = K$$

where K is some positive, finite constant.
Taylor Expansions
Another approach to figuring out rates of convergence of functions is to think about **Taylor's Series Approximations**
Recall that if you have a function $f(x)$ near a point $x = a$ and $f(x)$ is infinitely-differentiable, you can write down

$$f(x) =$$

or you can truncate this series and write down

$$f(x) \approx$$

Exercise
Write down the following Taylor Series Approximations (for small h):

$\sin(h) \approx$

$\cos(h) \approx$

$e^h \approx$

$(1 + h)^p \approx$

Example
We can use this new way to re-do our previous exercises and how that $\cos(h) + \frac{h^2}{2} = 1 + O(h^4)$

What is the rate of convergence of $\sin(h^3)$ as $h \to 0$?