Math 370 Fall 98

Quiz 4

Numerical Analysis

Name: _____

Date: _____ Friday October 9
Time Begun: _____ Ron Buckmire
Time Ended: _____

Topic: Root-Finding Algorithms

The idea behind this quiz is for you to apply your knowledge of the standard root finding techniques to a new method. This is a test of applying knowledge to deal with a new situation.

Instructions:

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. This quiz is due on Wednesday October 14, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

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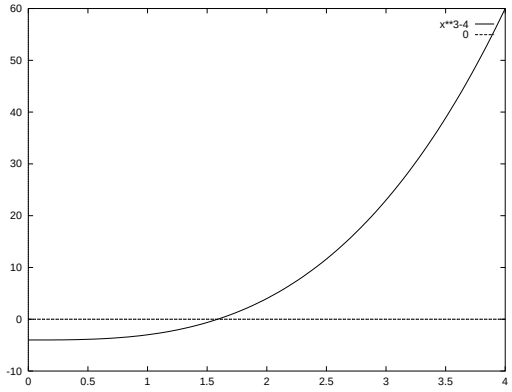
1. Consider a new method of finding a root of an equation, called The Lazy Newton or "Fixed Slope" method. This is similar to Newton's Method except that instead of taking the derivative at EVERY step, one computes the derivative once at the point of the initial guess p_0 and uses only that derivative in every subsequent iteration.

   The general formula for the Lazy Newton method is: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_0)}$

(a) [6 pts] Indicate on the graph of the function $f(x) = x^3 - 4$ below what the first few approximations to the root, p_1, p_2, p_3 will look like, using the Lazy Newton method, given that $p_0 = 4$. Make sure you indicate how you computed p_1, p_2 and p_3.



(HINT: Think about what approximations Newton's Method would produce and then think about how Lazy Newton's Method would be different.)

(b) [2 pts] Treat The Lazy Newton method as just any old Picard Iteration. For the given function of $f(x) = x^3 - 4$ with $p_0 = \alpha$ what is the iterative function $g(x)$ for the Lazy Newton method?

(c) [2 pts] If you are told that the initial guess is $p_0 = \alpha$ derive a condition on α for which you are confident that the Lazy Newton method will converge. (HINT: what is the condition on g for which you know Picard Iteration will always converge?)