Numerical Analysis

Math 370 Spring 2009 — MWF 11:30am - 12:25pm Fowler 110

 http://faculty.oxy.edu/ron/math/370/09/

Worksheet 17

SUMMARY Introduction to Approximation Theory
READING Recktenwald, Sec 9.1, pp 455–468; Mathews & Fink

Approximation Theory

In approximation theory we have a set of m data points (x_k, y_k) for which we do not know what the actual function $f(x)$ which reflects the relationship between the input variable x and the output y.

Suppose we define the **deviation** as $\delta_k = P(x_k) - y_k$ and find a function $P(x)$ such that the total deviations between the function $P(x)$ and the data points (x_k, y_k) is minimised. There is more than one way to do this.

We define a function E which represents the total deviation we are trying to minimize and we want to find P which minimizes E, where E can have different forms.
Some Ways To Formulate E Are:

$$E_1 = \sum_{k=1}^{m} |P(x_k) - y_k|$$

OR

$$E_\infty = \max_{1 \le k \le m} |P(x_k) - y_k|$$

OR

$$E_2 = \sum_{k=1}^{m} [P(x_k) - y_k]^2$$

From statistics we know that if the data are **normally distributed** then the square error $E = (E_2)^2$ is the best form of the error to use to measure how well $P(x)$ is approximating the unknown function $f(x)$ represented by the data y_k.

Linear Fit
If we assume that the polynomial we choose for $P(x)$ is linear so that $P(x) = ax + b$ then the problem of finding P becomes a minimization problem. If we consider E is a function of the parameters a and b what is the problem we have to solve, mathematically?

EXAMPLE

Let's find the minimum of $E(a, b)$.

Therefore

$$a = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - \overline{x}^2} \qquad b = \frac{\overline{x^2} \cdot \overline{y} - \overline{xy} \cdot \overline{x}}{\overline{x^2} - \overline{x}^2}$$

The line $P(x) = ax + b$ is known as the "least squares" line, or "line of best fit" or "regression line"

EXAMPLE

Consider the following data. We shall compute the **line of best fit** for the data and sketch it on the graph paper on the next page. You may try drawing what looks like a line of best fit by "eye" in one ink color and seeing how that compares with the computed regression line in a different ink color.

x_i	y_i	x_i^2	$x_i y_i$	$P(x_i)$	$\|y_i - P(x_i)\|$
1	1.3				
2	3.5				
3	4.2				
4	5.0				
5	7.0				
6	8.8				
7	10.1				
8	12.5				
9	13.0				
10	15.6				

It would suck if we had to do these calculations by hand. Let's try and use MATLAB as our calculator! (Write down the commands you use here)

RECALL

$$\text{slope} = \frac{\overline{xy} - \overline{x}\cdot\overline{y}}{\overline{x^2} - \overline{x}^2} \qquad \text{intercept} = \frac{\overline{x^2}\cdot\overline{y} - \overline{xy}\cdot\overline{x}}{\overline{x^2} - \overline{x}^2}$$

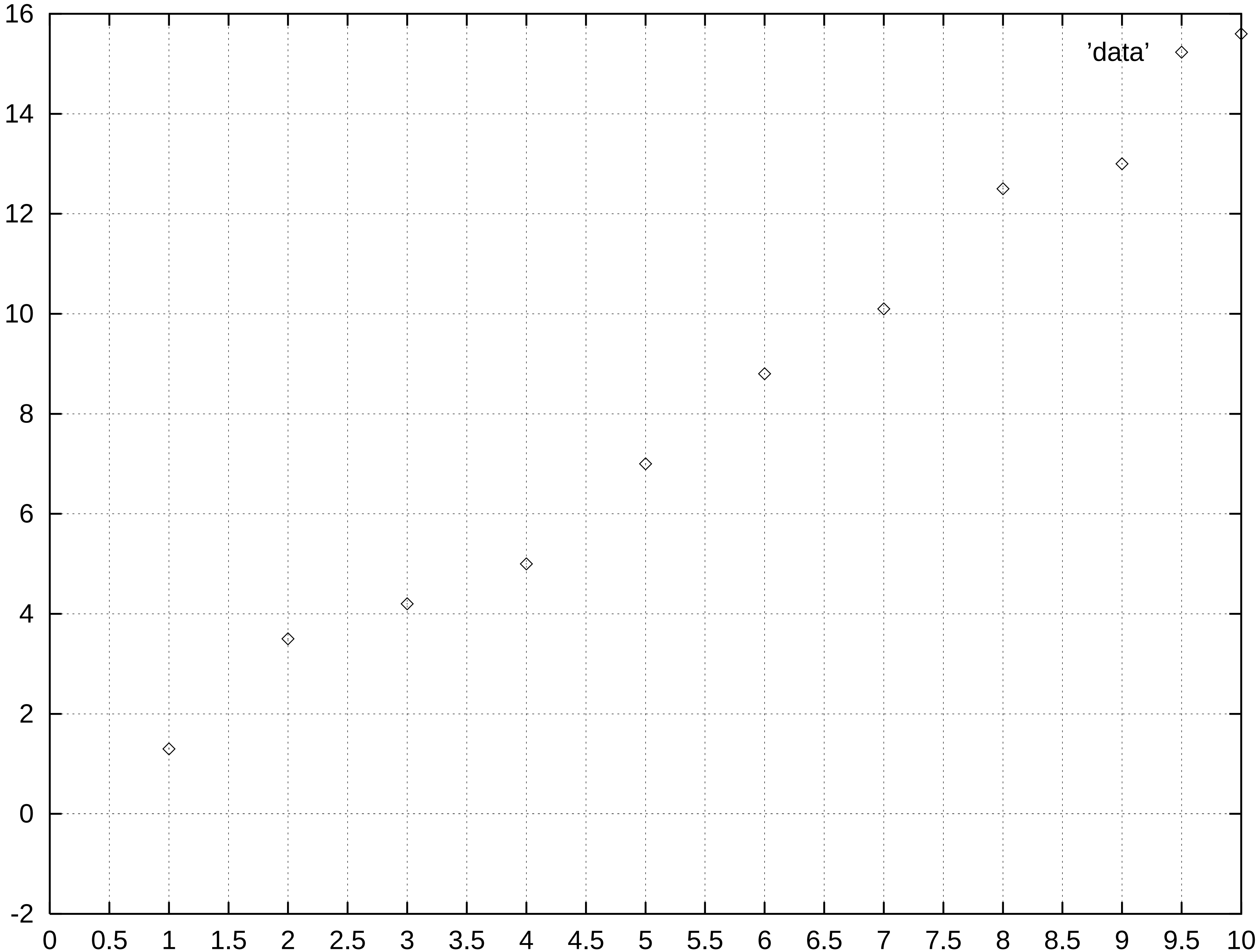
Using MATLAB to compute the line of best fit

Given a vector of inputs in **x** and **y** MATLAB will compute the slope and intercept of the line of best fit using the `linefit` command. You can find **linefit** in the **NMM** Toolbox `S:\Math Courses\Math370\Spring2009\NMM` under the `fit` directory.

GROUPWORK

Use `linefit` to find the line of best fit for the above data. (Write down the commands you use here)

Use the `plot` and `linspace` commands to plot the original data and the line of best fit on the same graph.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