QUIZ 5 Numerical Analysis

**Name**: ______________________________

**Friday March 20**
Prof. Ron Buckmire

Time Begun: ____________________
Time Ended: ____________________

## Topic : Nonlinear Systems of Equations

TThe idea behind this quiz is for you to obtain more practice solving a non-linear system of equations. Specifically, I want you to show that you can calculate using Successive Substitution, Seidel Iteration or Newton's Method.

## Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

## Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/370/09/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday March 23**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

**Pledge:** I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. In class we found one of the points of intersection of the hyperbola $4x^2 - y^2 = 1$ and the circle $(x-1)^2 + y^2 = 2^2$ to be (1.1165151,1.9966032).
   Let $g_1(x,y) = \dfrac{8x - 4x^2 + y^2 + 1}{8}$ and $g_2(x,y) = \dfrac{2x - x^2 + 4y - y^2 + 3}{4}$ where $\vec{G}(\vec{x}) = \begin{bmatrix} g_1(x,y) \\ g_2(x,y) \end{bmatrix}$

(a) *[1 pt]* Show that the fixed point(s) of the vector function $\vec{G}(\vec{x})$ are exactly the points of intersection of the hyperbola $4x^2 - y^2 = 1$ and circle $(x-1)^2 + y^2 = 4$. (HINT: one way to do this is to show algebraically that the fixed points of $\vec{G}$ satisfy the exact same equation that the points of intersection do.)

(b) *[2 pts]* Starting with an initial guess of $\vec{x}_0 = (1,2)^T$ compute the next approximation to the fixed point of $\vec{G}$ using Successive Substitution, $\vec{x}_k = \vec{G}(\vec{x}_{k-1})$

(c) *[2 pts]* Starting with an initial guess of $\vec{x}_0 = (1,2)^T$ compute the next approximation to the fixed point of $\vec{G}$ using Seidel Iteration.

(d) *[2 pts]* Considering $\vec{f}(\vec{x}) = \begin{bmatrix} 4x^2 - y^2 - 1 \\ (x-1)^2 + y^2 - 2^2 \end{bmatrix}$ Find the Jacobian matrix $J(x,y)$ for the system.

(e) *[3 pts]* Starting with an initial guess of $\vec{x}_0 = (1,2)^T$ compute the next approximation to the fixed point of $\vec{G}$ (which is also the root of $\vec{f}$) using Newton's Method.

*You may have to attach/staple an extra sheet with your calculations on it to support your answers.*