Numerical Analysis

Math 370 Spring 2009 MWF 11:30am - 12:25pm Fowler 110

 `http://faculty.oxy.edu/ron/math/370/09/`

Homework Set 1

12 questions, 50 points

ASSIGNED: Mon Jan 26 2009 **DUE: Wed Feb 04 2009**

1. (*2 points*) Use three-digit rounding **and 3 digit chopping** to perform the following calculations. Write your answers as 3-digit decimal floating point numbers of the form $.d_1d_2d_3E \pm N$. **(a)** $(121 - 0.327) - 119$ **(b)** $(121 - 119) - 0.327$

2. (*2 points*) *Mathews*, **#4, page 23.** Convert the following binary numbers to decimal (base 10) form. **(a)** 1.0110101_2 **(b)** 11.0010010001_2

3. (*2 points*) *Mathews*, **#5, page 23.** The numbers in the previous question are approximately $\sqrt{2}$ and π, respectively. Find the absolute and relative error in these binary approximations. **(a)** Use $\sqrt{2} = 1.41412135623709...$) **(b)** Use $\pi = 1 = 3.14159265358979...$)

4. (*8 points*) Find the limits and the rates of convergence of the following functions to those limit in terms of $\mathcal{O}$ as $h \to 0$
(a) $\dfrac{\sin(h) - h\cos(h)}{h}$ **(b)** $\dfrac{1 - e^{h^2}}{h^2}$ **(c)** $\dfrac{\tan(h)}{h}$ **(d)** $\dfrac{1 - \cos(h)}{h}$

5. (*3 points*) *Mathews*, **#1, page 37**. Find the error E_{abs}, relative error E_{rel} and determine the number of significant digits in each approximation.
(a) $x = 2.71828182$, $\tilde{x} = 2.7182$ **(b)** $y = 98,350$, $\tilde{y} = 98,000$ **(c)** $z = 0.000068$, $\tilde{z} = 0.00006$

6. (*4 points*) *Mathews*, **#10, page 38**. Given $e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + \mathcal{O}(h^5)$ and $\sin(h) = h - \frac{h^3}{3!} + \mathcal{O}(h^5)$ determine the order of approximation of the sum and product of the two functions.

7. (*3 points*) The sequence $\{F_n\}$ described by $F_0 = 1$, $F_1 = 1$, and $F_{n+2} = F_n + F_{n+1}$, if $n \geq 0$ is called the *Fibonacci sequence*. Consider the sequence $\{x_n\}$, where $x_n = F_{n+1}/F_n$. Assuming that the limit $\lim_{n\to\infty} x_n = x$ exists, show that the limit of the ratio of consecutive terms of the Fibonacci sequence is $x = (1 + \sqrt{5})/2$. This number is called the *golden ratio*.

8. (*4 points*) *Recktenwald*, **#2, page 77.** Evaluate the following quantities by using built-in MATLAB functions:
(a) $\cosh(5)$ **(b)** $\sinh(-2)$ **(c)** $(e^5 + e^{-5})/2$ **(d)** $\text{erf}(1.2)$ **(e)** $\beta(1, 2)$ **(f)** $\beta(0.4, 0.7)$
(g) $J_0(2)$ **(h)** $Y_0(2)$

9. (*4 points*) *Recktenwald*, **#3, page 77.** Use colon notation to create vectors identical to those produced by the following MATLAB commands. Use the `norm` command to show that the vectors are identical *without* printing the elements.
(a) `x = linspace(0,10,5)` **(b)** `x = linspace(-5,5)` **(c)** `x = logspace(1,3,3)`
(d) `x = logspace(1,3,5)`

10. (*3 points*) *Recktenwald*, **#24, page 81.** Plot $\sin\theta$ versus θ for 60 points in the interval $0 \le \theta \le 2\pi$. Connect the point with a dashed line *and* label the points with open circles.

11. (*5 points*) *Recktenwald*, **#27, page 81.** Write the MATLAB statements to create a plot of $y = \text{erf}(\alpha x)$ for $0 \le x \le 5$ and $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9, 1.1$. Arrange the plot so that x is on the horizontal axis, and different curves correspond to α values. Choose 100 x values so the curves look smooth.

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(*10 points*) Use a separate sheet of paper to discuss your understanding of how the fact that computers have fixed amount of memory to represent floating point numbers causes different kinds of errors. Write at least three (3) paragraphs. In particular, give *your own* understanding of the terms **overflow**, **round-off error** and **mantissa**. What was the numerical precision of the machine you did your calculations on for this homework set? Explain how you know what the precision of your computing device is.

Self-Assessment: In addition, write at least one paragraph describing how you approached this homework set, and what you found most challenging, and least challenging about it. You can also use this space to give me feedback on any other part of the course (quizzes, class-time, website, grading) that you wish.
The point of this part of the homework is to give you an opportunity to reflect on your own learning in a more thoughtful and reflective way. This is known as a **formative assessment**.

BONUS (10 points)

(a.) How many multiplications and additions are required to determine a sum of the form $\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{i} a_i b_j$?

(b.) Modify the sum in part **(a)** to an equivalent form that reduces the number of computations.

(c.) Write a MATLAB function m-file which executes both forms of the sum and outputs the number of operations for each form of the sum for an inputted value of n.

NOTES

This homework sets is due at 5pm on **Wednesday February 4**. You are **strongly** encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work. You should indicate on your **neatly-written** solution manuscripts which students you collaborated with. If you encounter difficulty, you should ask questions on the online message board at `http://moodle.oxy.edu` , or via the *Numerical Analysis* class email list at `math370-L@oxy.edu`, or come see me in my office. Make sure your name appears on each piece of paper you hand in, all pages are stapled together, and restate the problem before solving it (this last requirement can be satisfied by attaching a copy of the HW assignment sheet.)