Numerical Analysis

Math 370 Fall 2004 — MWF 2:30 - 3:25pm

 — Fowler North 5

Homework Set 3

5 questions+journal= 40+10 points

ASSIGNED: Fri Oct 29 2004 **DUE: Fri Nov 12 2004**

1. (*8 points*) Show that the following identities are true for all n dimensional vectors $\vec{x}$

(a.) Show that $|\vec{x}|_\infty \leq |\vec{x}|_2 \leq |\vec{x}|_1$

(b.) If $\vec{a} = (1, -2, 4, -8, 16)^T$ and $\vec{b} = (1, 1, 1)^T$ show that the vectors $\vec{a}$ and $\vec{b}$ satisfy the identity from part (a).

2. (*8 points*) Recall that in Calculus the following integral could be found by the technique of partial fractions

$$\int \frac{x^2 + x + 1}{(x-1)(x-2)(x-3)^2(x^2+1)} dx$$

This would involve finding the coefficients A_i for $i = 1, 2, \ldots, 6$ in the expression

$$\frac{x^2 + x + 1}{(x-1)(x-2)(x-3)^2(x^2+1)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-2)} + \frac{A_3}{(x-3)^2} + \frac{A_4}{(x-3)} + \frac{A_5 x + A_6}{x^2+1}$$

Write down the system of equations necessary to find the unknown coefficients and then use MATLAB to solve the system and find the anti-derivative.

3. (*8 points*) Consider the non-linear system

$$\begin{aligned} 0 = f_1(x, y) &= x^2 + y^2 - 2 \\ 0 = f_2(x, y) &= xy - 1 \end{aligned}$$

(a.) Verify that the solutions are $(1, 1)$ and $(-1, -1)$

(b.) Sketch a graph of the functions to indicate the points of intersection

(c.) What difficulties arise if you try to use Newton's Method for Systems to find the solutions?

(d.) Use a different numerical method to find both solutions to the nonlinear system.

4. (*8 points*) Consider the system

$$\begin{aligned} 5x - y &= 4 \\ -x + 5y + -2z &= 2 \\ -2y + 5z &= 3 \end{aligned}$$

(a.) Use Gauss-Seidel and Jacobi Iteration to solve the system using a tolerance of 10^{-8}

(b.) Mathematically explore which value of ω solves the system the fastest using SOR.

(c.) Show that the given system has a Jacobi matrix (i.e. $D^{-1}(L+U)$) which has a spectral radius of $\frac{1}{\sqrt{5}}$.

(d.) Use your result in (c) to confirm that the theoretical optimal relaxation parameter for SOR is 1.05573.

5. (*8 points*) Linear systems are just a special case of nonlinear systems. Let's see what happens if we use nonlinear solvers on linear systems.

(a.) Rewrite the previous system so that it is a fixed point problem. (HINT: just change the system to be $A\vec{x} - \vec{b} + \vec{x} = \vec{G}(\vec{x})$.)

(b.) Use Successive Substitution and Seidel iteration to find the solution of the system (i.e. the fixed point of $\vec{G}$) to within a tolerance of 10^{-8}

(c.) Use Newton's Method for System to also solve the same system to the same tolerance. Make sure you use the correct $\vec{F}(\vec{x})$ and Jacobian to ensure you are solving the same problem you solved in part (b).

Liberal use of the `diary` command together with printouts of any Matlab m-files you write are expected to be included.

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(*10 points*) Now that you have used Jacobi, Gauss-Seidel, Seidel, Successive Substitution, Newton's Method and SOR to solve the same problem discuss which method you prefer, and which method seems the most useful when faced with solving a system of multiple equations in multiple variables. Would it matter if it were a nonlinear or linear system? Do you see any relationships or similarities between the methods?

Self-Assessment: In addition, write a paragraph describing your expectations for the term project. The Term Project will most likely involve applying the numerical methods you now know to a new mathematical domain you have not seen before. How will you work with your partner to ensure that a fair division of work occurs? How would you like to have your Term Project partner chosen? By you or by me?

NOTES

You are **strongly** encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work. You should indicate on your neatly-written solution manuscripts which students you collaborated with. If you encounter difficulty, you should ask questions on the online message board at `http://blackboard.oxy.edu` , or via the *Numerical Analysis* class email list at `math370-L@oxy.edu`, or come see me in my office.