Report on Test 2

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Point Distribution (N=13)

Range	92+	87+	80+	75+	70+	65+	60+	50+	50-
Grade	A	A-	B+	B	B-	C+	C	C-	F
Frequency	5	3	0	2	1	0	1	0	1

Comments

Overall Overall performance was "improved." The average was 81.8 (up 2 points). More than half the class got an A on the exam! There was a perfect score of 100 and two almost-perfect scores of 97.

#1 Functional Iteration. This problem was similar to Question #2 from Exam 1. The idea of recursive iteration is a very important one. Many, many numerical algorithms involve the idea of iterating where your next guess to the solution is a function of your previous guess, i.e. $x_{n+1} = g(x_n)$. Understanding how to tell when you are given such a scheme that it is actually solving the problem you want is important. In part (a), you need to show that the fixed point of the iteration scheme $g(x)$ is actually $\sqrt{R}$, i.e. $g(\sqrt{R}) = \sqrt{R}$. Part(b) is finding the derivative of a rational function $(f(x)/g(x))' = (f'g - fg')/g^2$. Part (c) involves you recalling that the order of convergence of a Piacrd iteration looks like $|e_{n+1}| = |g'(p)||e_n| + \frac{1}{2}|g''(p)||e_n|^2$ so that if $g'(p) = 0$ then the iteration scheme is superlinear. It turns out that this scheme is cubic! part(d) can be completed without doing part (c), it just involves actually executing the iteration scheme.

#2 Matrix Norms. This problem was similar to the latest Quiz. From understanding the definition of the matrix norms $||A||_1$, $||A||_\infty$ and $||A||_2 = \sqrt{\rho(A^TA)}$ and applying it for the special case of a diagonal matrix you were able to derive an important result that the norm of a diagonal matrix is just the magnitude of the largest eigenvalue. The main point here is realizing that $A^T = A^2$ and that the $\max_i |\lambda_i| = \sqrt{\max_i \lambda_i^2} = |\lambda_{max}|$

#3 Newton's Method for Nonlinear Systems. I probably should have split this question into pieces and ask you explicitly to find the Jacobian and its inverse but whenever you see "Newton's Method" you should either be looking to find $f'(x)$ or J. This problem is about thinking of Newton's Method for Systems as a special case of a successive substitution method in the same way that regular Newton's Method can be thought of as a functional iteration. Part (a) is mostly algebra. Part (b) can be done without knowing part (a) and involves plugging in the given function into the formula given in part (a). Part(c) is again about illustrating that can you exceute a given algorithm by hand (i.e. do the calculations the computer would do). I'm interested in seeing HOW you do the calculations, not the specific numerical answer.