Differential Equations

Math 341 Fall 2014 | MWF 3:00-3:55pm Fowler 307

 | http://faculty.oxy.edu/ron/math/341/14/

Worksheet 18

TITLE Linear Systems with Real Eigenvalues
CURRENT READING Blanchard, 3.3

Homework #7 Assignments due Monday October 27 (* indicates EXTRA CREDIT)
Section 3.3: 3, 4, 7, 8, 20*.
Section 3.4: 1, 2, 3, 4, 16*, 23*.

SUMMARY
We'll continue to explore the various scenarios that occur with linear systems of ODEs that possess real eigenvalues.

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1. Two Real Eigenvalues

Given a system of linear ODEs with associated matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the characteristic polynomial $p(\lambda) = (a-\lambda)(d-\lambda) - bc = \lambda^2 - (a+d)\lambda + ad - bc = \lambda^2 - \mathrm{tr(A)}\lambda + \det(\mathrm{A}) = 0$.

EXAMPLE
What is the condition the characteristic polynomial $p(\lambda) = \lambda^2 - (a+d)\lambda + ad - bc$ must satisfy in order to produce real eigenvalues?

2. Classifying Equilibrium Points

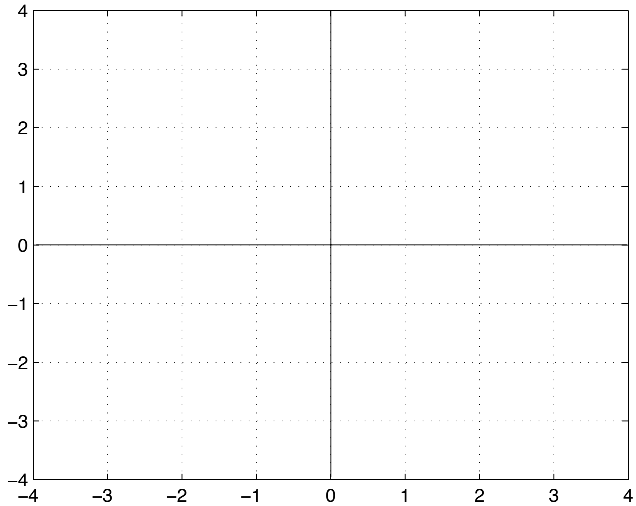
Suppose a linear system has two real, nonzero, distinct eigenvalues λ_1 and λ_2.

The solutions λ_1 and λ_2 to the characteristic polynomial can be classified into a number of different cases depending on the qualities the eigenvalues possess. In addition, the equilibrium at the origin can be classified as well, and a typical phase portrait sketched for each case.

GroupWork
On the next few pages, you will find a specific case, with an example of an ODE that satisfies the case and a classification of the origin. For each case:
Check that the given matrix will definitely produce real eigenvalues. Then find the eigenvalues and eigenvectors in order to write down the general solution of the given ODE.
Use `HPGSystemsSolver` or `PPLANE` to help you sketch the phase portrait for each case on the given axes. Also sketch the nullclines. Write down a few sentences describing your observations of the phase portrait.

3. CASE 1: $\lambda_1 > 0$ and $\lambda_2 > 0$

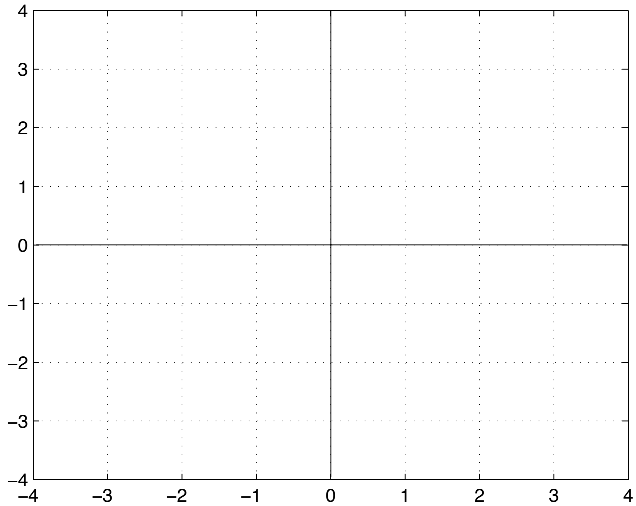
In this case the origin is an **unstable source**.



Solve $\dfrac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \vec{x}$.

4. **CASE 2:** $\lambda_1 < 0$ **and** $\lambda_2 < 0$

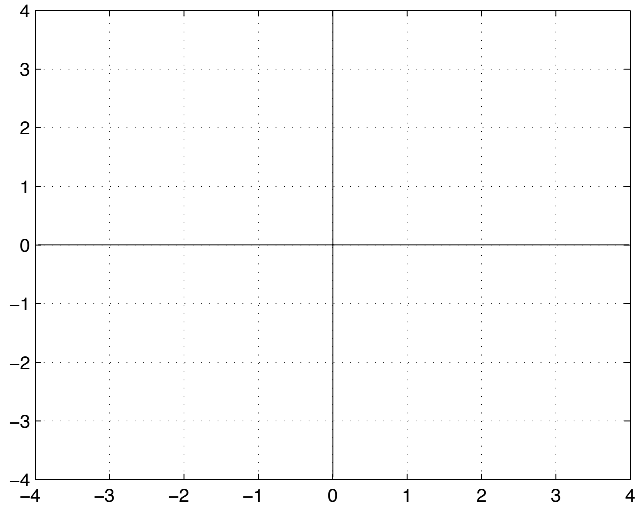
In this case the origin is a **stable sink**.



Solve $\dfrac{d\vec{x}}{dt} = \begin{bmatrix} -2 & -1 \\ 2 & -5 \end{bmatrix} \vec{x}$

5. CASE 3: $\lambda_1 > 0$ and $\lambda_2 < 0$

In this case the origin is a **unstable saddle**.



Solve $\dfrac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{x}$