Differential Equations

Math 341 Fall 2014 MWF 3:00-3:55pm Fowler 307
 http://faculty.oxy.edu/ron/math/341/14/

Class 2: Friday August 29

TITLE Separation of Variables
CURRENT READING Blanchard, §1.2 and §1.3

Homework Assignments due Friday September 5
Section 1.1: 2, 3, 13,14.
Section 1.2: 1, 2, 3, 6, 21, 27, 32.
Section 1.3: 7, 8, 12, 13,14,16.

SUMMARY
In today's class we shall review an analytical technique for solving a particular class of first-order (separable) ODEs known as **Separation of Variables**.

---

1. Solving Separable Differential Equations

DEFINITION: **separable DE**

A **separable first-order differential equation** is one which has the form $\frac{dy}{dx} = g(x)h(y)$

The technique for solution is to separate the variables in the equation by placing everything with an independent variable on one side, and everything with a dependent variable on the other. This produces:

$$\frac{dy}{h(y)} = g(x)dx$$

One can then treat each side of the equation as an indefinite integral,

$$\int \frac{dy}{h(y)} = \int g(x)dx$$

which, if each function $1/h(y)$ and $g(x)$ have anti-derivatives $H(y)$ and $G(x)$, respectively produces

$$H(y) = G(x) + C$$

The above equation thus defines (implicitly) a family of solutions to the given first-order DE. When an initial condition $y(a) = b$ is also given, then a particular solution can be obtained.

EXAMPLE
Let's consider the Malthusian Model of population $P' = kP, P(0) = P_0$ and obtain the solution by separation of variables.

NOTE: k is a parameter in the Malthusian population model, which has P as a **dependent** variable and t as an **independent variable**.

Exercise

Let's consider the Verhulst or Logistic Model of Population $P' = kP(1 - P/N), P(0) = P_0$. What are the interpretation of the parameters k and N in the Verhulst model?

GroupWork

Show that if you make the change of variables $Q(t) = P(t)/N$ the Logistic Model can be written as $Q' = kQ(1 - Q)$ which has a solution of the form $Q(t) = \dfrac{1}{1 + Ce^{-kt}}$ where C is any real number.