Test 1: Differential Equations

Math 341 Fall 2013 — Friday October 18

 12:50-1:45pm

Name: ______________________________

Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. This is a 55-minute, no-notes, closed book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences as much as possible and CLEARLY indicate your final answers to be graded from your "scratch work."

Pledge: I, ______________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		20
2		15
3		15
BONUS		5
Total		**50**

1. *[20 points total.]* **Existence and Uniqueness Theorem, Functions, Equilibrium Solutions, Separation of Variables, Interval of Validity.** ANALYTIC & VERBAL.

Determine whether the following statements are **TRUE** or **FALSE** and place your answer in the box. To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! The explanation for your answer is worth FOUR POINTS while your TRUE or FALSE answer is worth 1 point.

1(a) *[5 points].* **TRUE or FALSE**: "The initial value problem $y' = 2|t|, \ y(0) = B$ has an equilibrium solution $y(t) = 0$ when $B = 0$."

$\boxed{}$

1(b) *[5 points].* **TRUE or FALSE**: "The initial value problem $y' = 2|t|, \ y(0) = B$ has a solution $y(t) = t^2 + B$ with interval of validity $t \geq 0$ for all values of B."

$\boxed{}$

1(c) *[5 points].* **TRUE or FALSE**: "The initial value problem $y' = 2|t|,\ y(0) = B$ has a unique solution for all values of B."

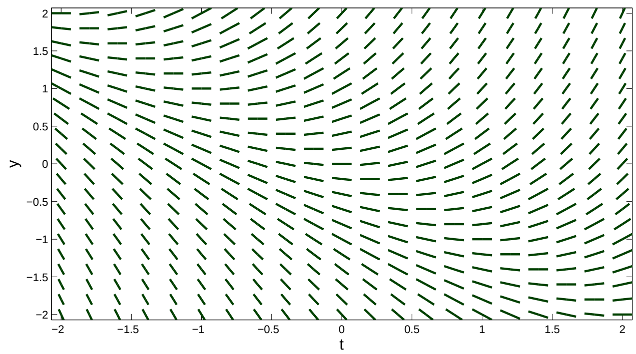
1(d) *[5 points].* **TRUE or FALSE**: "The initial value problem $y' = 2|t|,\ y(0) = B$ has a solution that is continuous and differentiable at $t = 0$ for all values of B."

2. *[15 points total.]* **Slope Fields, Solution Techniques for Linear ODEs, Autonomous DEs, Non-homogeneous DEs.** ANALYTIC, VERBAL & VISUAL.

2(a) *[5 points].* Find the general solution of **(ODE A)** $\dfrac{dy}{dt} = y + t$.
STATE YOUR SOLUTION TECHNIQUE (MUST BE DIFFERENT FROM PART(B)).

2(b) *[5 points].* Find the general solution of **(ODE B)** $\dfrac{dy}{dt} = y + 1$.
STATE YOUR SOLUTION TECHNIQUE (MUST BE DIFFERENT FROM PART (A))

2(c) *[5 points].* Mark whether the slope field below corresponds to **(ODE A)** or **(ODE B)** and EXPLAIN YOUR REASONING.



3. *[15 pts. total]* **Phase Lines, Equilibria, Bifurcations, Geometric Representations.** VISUAL & ANALYTIC.

Consider the following differential equation: $\dfrac{dy}{dt} = y^2 - \alpha^2$, where α is a real-valued parameter.

3(a) *[5 points]* What are the equilibrium values of the differential equation? Identify them as y^*. They should depend on values of the parameter α.

3(b) *[5 points]* Is there a bifurcation value for the parameter α? If so, call it α_B and draw phase lines corresponding to the cases where $\alpha < \alpha_B$, $\alpha = \alpha_B$ and $\alpha > \alpha_B$. Indicate locations (and values) of all nodes, sinks or sources y^* on your phase lines.

3(c) *[5 points]* Draw a bifurcation diagram for the differential equation $\dfrac{dy}{dt} = y^2 - \alpha^2$ in the αy^*-plane. Indicate clearly on the graph where sinks, sources and nodes occur, if they exist.

BONUS. *[5 points]* Solve $y'' + 3y' + 2y = 0$ where $y(0) = 1, y'(0) = 0$.