Differential Equations

Class 2: Friday September 3

TITLE Separation of Variables
CURRENT READING Blanchard, §1.2 and §1.3

Homework Assignments due Friday September 10
Section 1.2: 1, 2, 3, 6, 25, 27, 32.
Section 1.3: 7, 8, 9, 10, 12, 15.

SUMMARY
In today's class we shall review an analytical technique for solving a particular class of first-order (separable) ODEs known as **Separation of Variables**.

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1. Solving Separable Differential Equations

DEFINITION: **separable DE**

A **separable first-order differential equation** is one which has the form $\dfrac{dy}{dx} = g(x)h(y)$

The technique for solution is to separate the variables in the equation by placing everything with an independent variable on one side, and everything with a dependent variable on the other. This produces:

$$\frac{dy}{h(y)} = g(x)dx$$

One can then treat each side of the equation as an indefinite integral,

$$\int \frac{dy}{h(y)} = \int g(x)dx$$

which, if each function $1/h(y)$ and $g(x)$ have anti-derivatives $H(y)$ and $G(x)$, respectively produces

$$H(y) = G(x) + C$$

The above equation thus defines (implicitly) a family of solutions to the given first-order DE. When an initial condition $y(a) = b$ is also given, then a particular solution can be obtained.

EXAMPLE
Let's consider the Malthusian Model of population $P' = kP, P(0) = P_0$ and obtain the solution by separation of variables.

Exercise

Let's consider the Verhulst or Logistic Model of Population $P' = kP(1 - P/N), P(0) = P_0$. Can you obtain a solution by the method of separation of variables?

DEFINITION: **dynamical system**

A dynamical system is a mathematical concept which has a current state and an evolution rule which determines how future values of the state are calculated from previous values of the state. There are both continuous dynamical systems (of which initial value problems are a particular example) and discrete dynamical systems (see chapter 8 of text).

$$y = \frac{1}{x}, x > 0$$

$$y = \frac{1}{x}, x \neq 0$$

$$y = 0, x \in \mathbf{R}$$

$$y = 0, x > 0$$