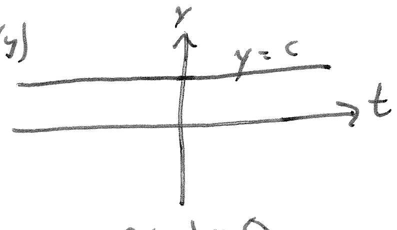
Math 341 Fall 2010 **SHOW ALL YOUR WORK** Quiz **Three**

1. (*2 points.*) What is an equilibrium solution for a 2-dimensional system of differential equations $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x})$ and how does it differ (geometrically) from an equilibrium solution for a differential equation, $\frac{dy}{dt} = f(y)$? (HINT: DRAW PICTURES REPRESENTING THE DIFFERENT SITUATIONS AND USE WORDS TO DESCRIBE THEM.)

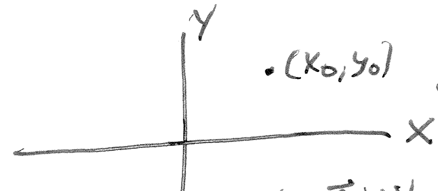
$y' = f(y)$



$f(c) = 0$

In 1-D equilibrium is a line

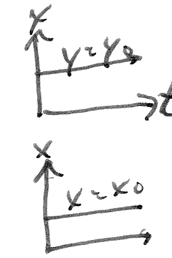
In 2-D it is a point (on graphs of 2 lines)



$\vec{x}' = \vec{f}(\vec{x})$

$\vec{x} = (x_0, y_0)$

or



2. (*2 points.*) What are the equilibrium solutions for the standard Lotka-Volterra predator-prey model $R' = aR - bRF$, $F' = -cF + dRF$? What is the physical interpretation of these equilibrium values on the predator F and prey R populations?

$0 = aR - bRF \Rightarrow R(a - bF) = 0 \Rightarrow R = 0$ or $F = a/b$

$0 = -cF + dRF \quad F(-c + dR) = 0 \quad F = 0$ or $R = c/d$

$(R, F) = (0, 0)$ and $(R, F) = (c/d, a/b)$ are equilibrium solutions

$R = F = 0$ is the trivial solution $\Rightarrow$ no rabbits or foxes means no change

$R = c/d$, $F = a/b$ means there exists values where the two co-exist in harmony

3. (*2 points.*) Explain in your own words what the difference between coupled and decoupled systems of equations are. Give an example of each type (linear, first-order, ordinary).

decoupled equations are autonomous in a single variable (other dependent variables do not appear) much easier to solve i.e. $x' = x, y' = y$

coupled: rate of change of one variable depends on another dependent variable e.g. $x' = y$, $y' = -x + y$

4. (*2 points.*) What is a reasonable guess for the general solution of $y' = -3y + t$? [HINT: How many unknown constants should your solution have?]

$y(t) = y_h + y_p = Ae^{-3t} + Bt + C$ Should have only one unknown constant. Guess: $\boxed{y = Ae^{-3t} + \frac{1}{3}t - \frac{1}{9}}$

$y' = -3Ae^{-3t} + B = -3(Ae^{-3t} + Bt + C) + t$

$(-3A + 3A)e^{-3t} + 1 \cdot (B + 3C) + t(3B - 1) = 0 \Rightarrow B = 1/3$

$B + 3C = 0 \Rightarrow C = -\frac{B}{3} = -\frac{1}{9}$

5. (*2 points.*) **TRUE or FALSE:** "Euler's Method can never be used to approximate solutions to a second-order nonlinear ordinary differential equation." EXPLAIN YOUR ANSWER.

False. Write the 2^{nd} order ODE as a 2-D system of ODEs

$y'' = f(x, y, y')$

Given: $y(0) = A$

$y'(0) = B$

Let $u = y$

$v = y'$

$v' = f(x, u, v)$

$u' = v$

$\vec{x} = \binom{u}{v}$

$u(0) = A$

$v(0) = B$

$\frac{d}{dt}\vec{x} = \vec{F}(\vec{x}) = \binom{f(x, u, v)}{v}$