BONUS QUIZ 3

DIFFERENTIAL EQUATIONS

Name: ____________________________

Friday December 3
Prof. Ron Buckmire

Topic : Advanced Laplace Transforms

The idea behind this quiz is to provide you with an opportunity to demonstrate your ability with inverting Laplace Transforms of a complicated function.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/10/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This bonus quiz is due on Monday December 6**, at the beginning of class.
   NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. We're interested in finding the function $f(t)$ whose Laplace Transform is

$$F(s) = A(s) - B(s) = \frac{1}{s^2} - \frac{e^{-s}}{s(1-e^{-s})}, \quad s > 0$$

(a) *2 points.* Compute $\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = a(t)$.

(b) *2 points.* If one considers $\frac{1}{1-e^{-s}}$ as the sum of a geometric series $\sum_{k=0}^{\infty} ar^k$ with first term $a = 1$ and ratio $r = e^{-s}$ then show that $\frac{e^{-s}}{s(1-e^{-s})}$ can be written as $\sum_{k=1}^{\infty} \frac{e^{-ks}}{s} = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \ldots$

(c) *3 points.* Recall that $\mathcal{L}^{-1}\left[e^{-as}F(s)\right] = f(t-a)\mathcal{H}(t-a)$. Using the result given in (b), compute $\mathcal{L}^{-1}\left[\frac{e^{-s}}{s(1-e^{-s})}\right] = b(t)$.

(d) *3 points.* Give a sketch of $a(t)$, $b(t)$ and $f(t) = a(t) - b(t)$ below for $t > 0$ (Use different pairs of axes for each graph.)