Math 341 Fall 2010 **SHOW ALL YOUR WORK** **Bonus Quiz Two**

1. Consider the system of ordinary differential equations

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

(a) *1 point.* Show that the matrix A has eigenvalues 0 and -1 and eigenvectors which are multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Write down the 2-parameter general solution of the system $\frac{d\vec{x}}{dt} = A\vec{x}$.

$$x(t) = c_1 e^{0\cdot t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= c_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = -1\begin{pmatrix} -2 \\ +1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1 - 2c_2 e^{-t} \\ c_2 e^{-t} \end{pmatrix}$$

(b) *2 points.* Find the exact solution $\vec{x}(t)$ for each of the trajectories which go through the points **A**$(1,1)$, **B** $(0,-2)$ and **C** $(4,0)$ at $t=0$.

$t=0,\ \vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 - 2c_2 \\ c_2 \end{pmatrix}$

$c_2 = 1 \Rightarrow c_1 = 1 + 2c_2 = 3$

$\vec{x}_A(t) = \begin{pmatrix} 1 - 2e^{-t} \\ 1e^{-t} \end{pmatrix}$

$= 3\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} -2 \\ 1 \end{pmatrix}e^{-t}$

$t=0,\ \vec{x}_B = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 0 \\ -2 \end{pmatrix} = \begin{pmatrix} c_1 - 2c_2 \\ c_2 \end{pmatrix}$

$c_2 = -2 \Rightarrow c_1 = 0 + 2c_2 = -4$

$\vec{x}_B(t) = \begin{pmatrix} -4 + 4e^{-t} \\ -2e^{-t} \end{pmatrix}$

$= -4\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 2e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$t=0,\ \vec{x}_C = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 - 2c_2 \\ c_2 \end{pmatrix}$

$c_2 = 0 \Rightarrow c_1 = 4 + 2c_2$

$c_1 = 4$

$\vec{x}_C = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(c) *2 points.* On the figure below clearly indicate the trajectories for each of the solutions which start at **A**$(1,1)$, **B** $(0,-2)$ and **C** $(4,0)$ ends up as $t \to \infty$. Label these endpoints **A**$'$, **B**$'$ and **C**$'$ respectively.