Math 341 Fall 2010 **SHOW ALL YOUR WORK** **Bonus Quiz One**

1. Consider the first-order, nonlinear, Clairault ordinary differential equation

$$y = x\left(\frac{dy}{dx}\right) - \frac{1}{4}\left(\frac{dy}{dx}\right)^2$$

(a) *1 point.* Confirm that the family of solutions is the set of **lines** $y = Cx - \frac{1}{4}C^2$.

$\frac{dy}{dx} = C$

$LHS = Cx - \frac{1}{4}C^2$ $\quad RHS = x(C) - \frac{1}{4}(C)^2$

$LHS = RHS$

(b) *3 points.* Show that the lines $y = Cx - \frac{1}{4}C^2$ are tangent to the curve $y = x^2$ at the point $\left(\frac{C}{2}, \frac{C^2}{4}\right)$ and sketch the curve and its tangents below for at least 4 values of C.

$y = x^2$

$\frac{dy}{dx} = 2x$

at $x = \frac{C}{2}$, $\frac{dy}{dx} = 2 \cdot \frac{C}{2} = C$ ✓

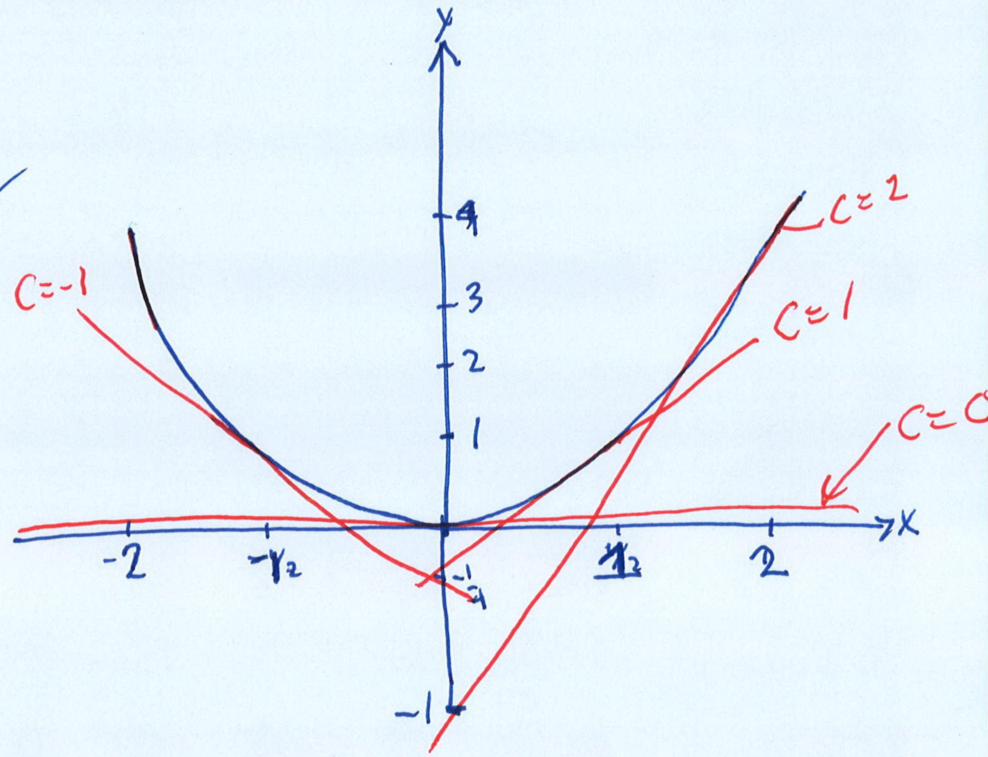
$C = 1$, $y = x - \frac{1}{4}$

$C = -1$ $y = -x - \frac{1}{4}$

$C = 0$ $y = 0$

$C = 2$ $y = 2x - \frac{4}{4}$

$C = -2$, $y = -2x - 1$



(c) *1 point.* Explain how parts (a) and (b) imply that $y = x^2$ is a singular solution of the given Clairault equation. [HINT: A singular solution of an ODE is one which solves the ODE but is not a member of the family of solutions.]

$y = x^2$

$\frac{dy}{dx} = 2x$

$LHS = x^2$

$RHS = x(2x) - \frac{1}{4}(2x)^2$

$= 2x^2 - \frac{1}{4}4x^2$

$= 2x^2 - x^2$

$= x^2$

$LHS = RHS$ ✓

$y = x^2$ is a solution since it is tangential to every member of the family of solutions given by $y = Cx - \frac{1}{4}C^2$

$y = x^2$ is called a SINGULAR solution