which is also separable. Separating variables, integrating, and solving for p yields

$$p(t) = \frac{-0.15\,ke^{3t/20}}{1 - 0.0133\,ke^{3t/20}}.$$

The solution with $p(0) = 30$ has $p(5) = 30$. Using $p(5) = 30$ as the initial condition, we get

$$p(t) = \frac{-0.15 \cdot 59\,e^{3t/20}}{1 - 0.0133 \cdot 59\,e^{3t/20}}$$

for $t \geq 5$. Using $p(5) = 28.1$ as the initial condition, we obtain

$$p(t) = \frac{0.15 \cdot 61.6\,e^{3t/20}}{1 - 0.0133 \cdot 61.6\,e^{3t/20}}.$$

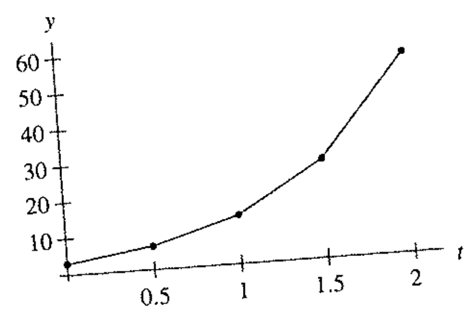
(d) The solution with $p(0) = 30$ is constant for $0 \leq t \leq 5$ because 30 is the carrying capacity. For $t > 5$, the solution decays toward a new, smaller equilibrium at $p = 11.25$. The solution with initial condition $p(0) = 20$ grows toward the carrying capacity until $t = 5$. Then it also decreases toward the new, smaller equilibrium at $p = 11.25$.

EXERCISES FOR SECTION 1.4

1.

Table 1.1
Results of Euler's method

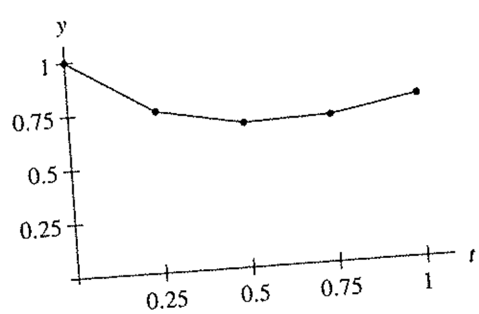
k	t_k	y_k	m_k
0	0	3	7
1	0.5	6.5	14
2	1.0	13.5	28
3	1.5	27.5	56
4	2.0	55.5	



2.

Table 1.2
Results of Euler's method (y_k rounded to two decimal places)

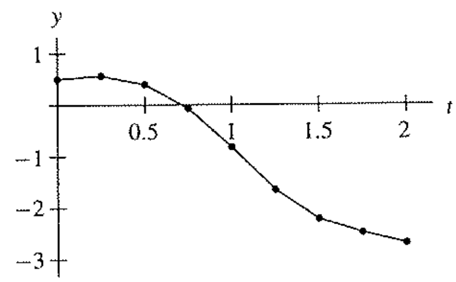
k	t_k	y_k	m_k
0	0	1	-1
1	0.25	0.75	-0.3125
2	0.5	0.67	0.0485
3	0.75	0.68	0.282
4	1.0	0.75	



3.

Table 1.3
Results of Euler's method (shown rounded to two decimal places)

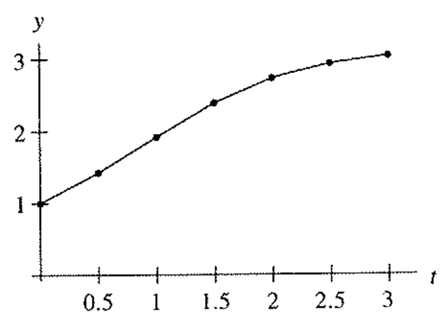
k	t_k	y_k	m_k
0	0	0.5	0.25
1	0.25	0.56	−0.68
2	0.50	0.39	−1.85
3	0.75	−0.07	−2.99
4	1.00	−0.82	−3.33
5	1.25	−1.65	−2.27
6	1.50	−2.22	−1.07
7	1.75	−2.49	−0.81
8	2.00	−2.69	



4.

Table 1.4
Results of Euler's method (to two decimal places)

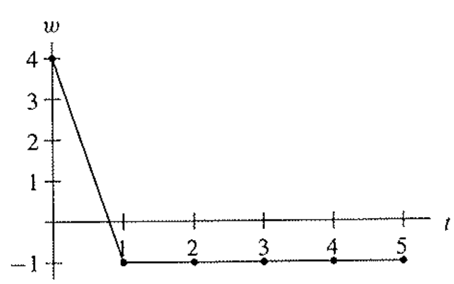
k	t_k	y_k	m_k
0	0	1	0.84
1	0.5	1.42	0.99
2	1.0	1.91	0.94
3	1.5	2.38	0.68
4	2.0	2.73	0.40
5	2.5	2.93	0.21
6	3.0	3.03	



5.

Table 1.5
Results of Euler's method

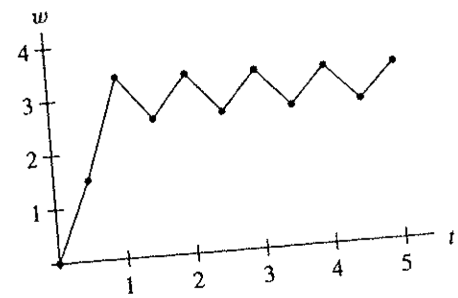
k	t_k	w_k	m_k
0	0	4	−5
1	1	−1	0
2	2	−1	0
3	3	−1	0
4	4	−1	0
5	5	−1	



6.

Table 1.6
Results of Euler's method (shown rounded to two decimal places)

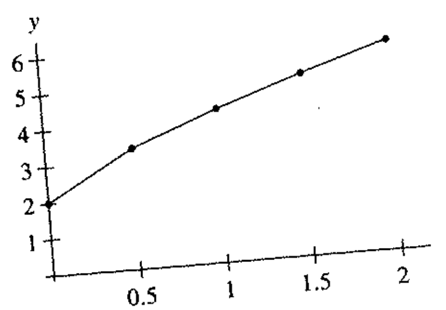
k	t_k	w_k	m_k
0	0	0	3
1	0.5	1.5	3.75
2	1.0	3.38	−1.64
3	1.5	2.55	1.58
4	2.0	3.35	−1.50
5	2.5	2.59	1.46
6	3.0	3.32	−1.40
7	3.5	2.62	1.36
8	4.0	3.31	−1.31
9	4.5	2.65	1.28
10	5.0	3.29	



7.

Table 1.7
Results of Euler's method (shown rounded to two decimal places)

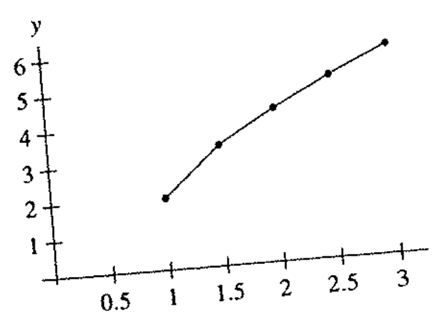
k	t_k	y_k	m_k
0	0	2	2.72
1	0.5	3.36	1.81
2	1.0	4.27	1.60
3	1.5	5.06	1.48
4	2.0	5.81	



8.

Table 1.8
Results of Euler's method (shown rounded to two decimal places)

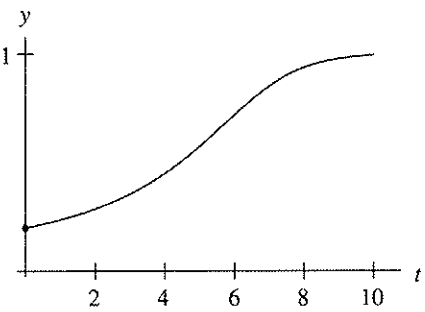
k	t_k	y_k	m_k
0	1.0	2	2.72
1	1.5	3.36	1.81
2	2.0	4.27	1.60
3	2.5	5.06	1.48
4	3.0	5.81	



9.

Table 1.9
Results of Euler's method (shown rounded to three decimal places)

k	t_k	y_k	m_k
0	0.0	0.2	0.032
1	0.1	0.203	0.033
2	0.2	0.206	0.034
3	0.3	0.210	0.035
⋮	⋮	⋮	⋮
99	9.9	0.990	0.010
100	10.0	0.991	



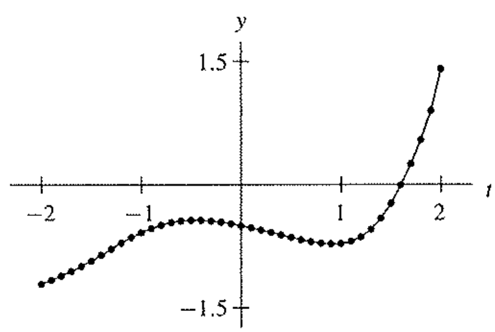
10.

Table 1.10
Results of Euler's method with Δt negative (shown rounded to three decimal places)

k	t_k	y_k	m_k
0	0	−0.5	−0.25
1	−0.1	−0.475	−0.204
2	−0.2	−0.455	−0.147
3	−0.3	−0.440	−0.080
⋮	⋮	⋮	⋮
19	−1.9	−1.160	0.488
20	−2.0	−1.209	0.467

Table 1.11
Results of Euler's method with Δt positive (shown rounded to three decimal places)

k	t_k	y_k	m_k
0	0	−0.5	−0.25
1	0.1	−0.525	−0.279
2	0.2	−0.553	−0.298
3	0.3	−0.583	−0.306
⋮	⋮	⋮	⋮
19	1.9	0.898	5.058
20	2.0	1.404	9.532



11. Because the differential equation is autonomous, the computation that determines y_{k+1} from y_k depends only on y_k and Δt and not on the actual value of t_k. Hence the approximate y-values that are obtained in both exercises are the same. It is useful to think about this fact in terms of the slope field of an autonomous equation.

12. Euler's method is not accurate in either case because the step size is too large. In Exercise 5, the approximate solution "jumps onto" an equilibrium solution. In Exercise 6, the approximate solution "crisscrosses" a different equilibrium solution. Approximate solutions generated with smaller values of Δt indicate that the actual solutions do not exhibit this behavior (see the Existence and Uniqueness Theorem of Section 1.5).

13. As the solution approaches the equilibrium solution corresponding to $w = 3$, its slope decreases. We do not expect the solution to "jump over" an equilibrium solution (see the Existence and Uniqueness Theorem in Section 1.5).

14.

Table 1.12
Results of Euler's method with $\Delta t = 1.0$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	1	2	1.41
2	2	3.41	1.85
3	3	5.26	2.29
4	4	7.56	

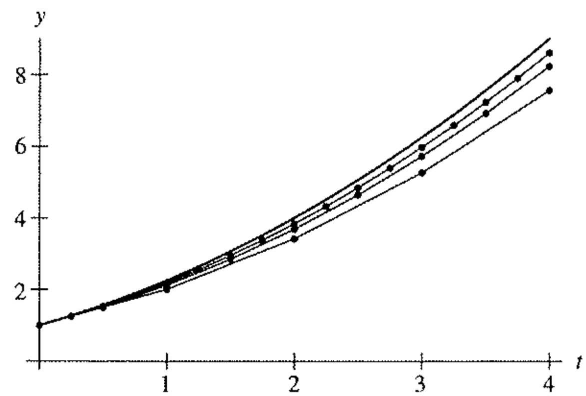
Table 1.13
Results of Euler's method with $\Delta t = 0.5$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	0.5	1.5	1.22
2	1.0	2.11	1.45
3	1.5	2.84	1.68
4	2.0	3.68	1.92
5	2.5	4.64	2.15
6	3.0	5.72	2.39
7	3.5	6.91	2.63
8	4.0	8.23	

Table 1.14
Results of Euler's method with $\Delta t = 0.25$ (shown to two decimal places)

k	t_k	y_k	m_k	k	t_k	y_k	m_k
0	0	1	1	9	2.25	4.32	2.08
1	0.25	1.25	1.12	10	2.50	4.84	2.20
2	0.50	1.53	1.24	11	2.75	5.39	2.32
3	0.75	1.84	1.36	12	3.0	5.97	2.44
4	1.0	2.18	1.48	13	3.25	6.58	2.56
5	1.25	2.55	1.60	14	3.50	7.23	2.69
6	1.50	2.94	1.72	15	3.75	7.90	2.81
7	1.75	3.37	1.84	16	4.0	8.60	
8	2.0	3.83	1.96				

The slopes in the slope field are positive and increasing. Hence, the graphs of all solutions are concave up. Since Euler's method uses line segments to approximate the graph of the actual solution, the approximate solutions will always be less than the actual solution. This error decreases as the step size decreases.



15.

Table 1.15
Results of Euler's method with $\Delta t = 1.0$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	1	2	0
2	2	2	0
3	3	2	0
4	4	2	

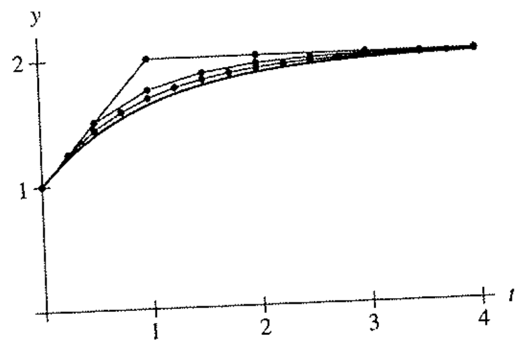
Table 1.16
Results of Euler's method with $\Delta t = 0.5$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	0.5	1.5	0.5
2	1.0	1.75	0.26
3	1.5	1.88	0.12
4	2.0	1.94	0.06
5	2.5	1.97	0.02
6	3.0	1.98	0.02
7	3.5	1.99	0.02
8	4.0	2.0	

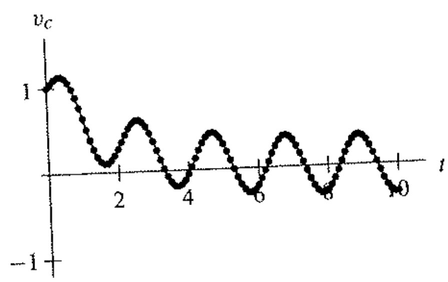
Table 1.17
Results of Euler's method with $\Delta t = 0.25$ (shown to two decimal places)

k	t_k	y_k	m_k	k	t_k	y_k	m_k
0	0	1	1	9	2.25	1.92	0.08
1	0.25	1.25	0.76	10	2.50	1.94	0.06
2	0.50	1.44	0.56	11	2.75	1.96	0.04
3	0.75	1.58	0.40	12	3.0	1.97	0.03
4	1.0	1.68	0.32	13	3.25	1.98	0.02
5	1.25	1.76	0.24	14	3.50	1.98	0.02
6	1.50	1.82	0.18	15	3.75	1.99	0.01
7	1.75	1.87	0.13	16	4.0	1.99	
8	2.0	1.90	0.10				

From the differential equation, we see that dy/dt is positive and decreasing as long as $y(0) = 1$ and $y(t) < 2$ for $t > 0$. Therefore, $y(t)$ is increasing, and its graph is concave down. Since Euler's method uses line segments to approximate the graph of the actual solution, the approximate solutions will always be greater than the actual solution. This error decreases as the step size decreases.

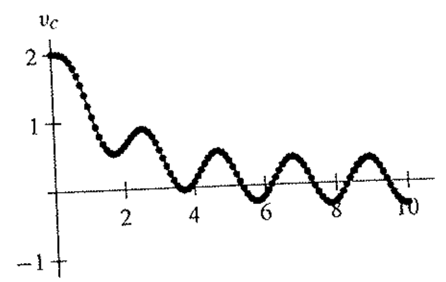


16.



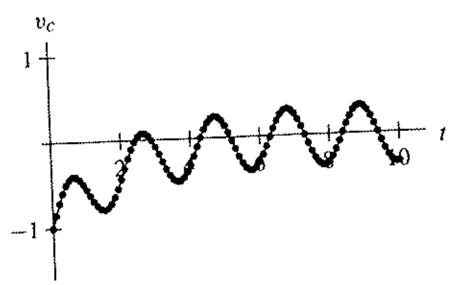
Graph of approximate solution obtained using Euler's method with $\Delta t = 0.1$.

17.



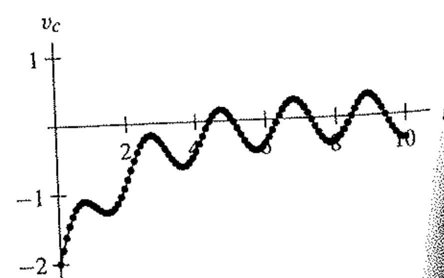
Graph of approximate solution obtained using Euler's method with $\Delta t = 0.1$.

18.



Graph of approximate solution obtained using Euler's method with $\Delta t = 0.1$.

19.



Graph of approximate solution obtained using Euler's method with $\Delta t = 0.1$.