# Differential Equations

Math 341 Fall 2009 ©2009 Ron Buckmire MWF 2:30-3:25pm Fowler 110 http://faculty.oxy.edu/ron/math/341/09/

## Class 4: Friday September 11

**TITLE** Euler's Method **CURRENT READING** Blanchard, 1.4

Homework Assignments due Friday September 18

Section 1.4: 5, 6, 13, 15 Section 1.5: 2, 3, 12, 14, 15. Section 1.6: 2, 7, 8, 19, 20, 30, 31, 41

#### SUMMARY

We will learn about a simple (and possibly familiar) numerical technique called Euler's Method which approximates solutions to ODEs quantitatively.

## Euler's Method

Given an expression for how the derivative of an unknown function y(x) changes, i.e. y' = f(x, y), and an initial value  $y(x_0) = y_0$  one can use Euler's Method to estimate y(x) at a point close by with bounded error.

$$y(x_{new}) = y(x_{old}) + \Delta y$$
 where  $\Delta y \approx y'(x_{old})\Delta x$ 

In other words

 $y_{new} \approx y_{old} + y'_{old}\Delta x$  and  $x_{new} = x_{old} + \Delta x$  or  $y_{k+1} \approx y_k + f(x_k, y_k)\Delta x$  and  $x_{k+1} = x_k + \Delta x$ . **Exercise** 

Draw a picture of the Euler approximation  $y_{new}$  starting at a point  $(x_{old}, y_{old})$ . Is the slope field involved?

#### EXAMPLE

#### Using Euler's Method To Approximate Solutions To Differential Equations

- 1. Suppose y changes with time t according to the equation y' = 1 + 2y.
- (a) What is the rate of change of y when y = 3?
- (b) Suppose when t = 0, y = 3. Use Euler's Method with  $\Delta t = .5$  to estimate y(1).
- (c) Is your estimate of y(1) an over-estimate or under-estimate?

To use Euler's Method generally the following table can be helpful

x	y	y'	$\Delta y$

### Slope Fields and Euler's Method Exercise

Consider the differential equation y' = -x/y with initial condition y(0) = 1. Given that the exact solution is  $y(x) = \sqrt{1-x^2}$ ,

(a) use the slope field to estimate y(1/2) for the solution that satisfies the given initial condition.

- (b) Compare your estimate with the exact value of y(1/2)
- (c) Use Euler's Method with  $\Delta x = .25$  to estimate y(1/2).
- (d) Is your Euler's Method estimate and over-estimate or under-estimate? Explain why.



To use Euler's Method generally the following table can be helpful

x	y	y'	$\Delta y$

# Numerical Error in Using Euler's Method

GROUPWORK

Complete the following sentences:

As the time step  $\Delta t$  \_\_\_\_\_\_ in magnitude, the numerical error in computing  $y(x_0)$  using Euler's Method decreases in magnitude.

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When y'' is \_\_\_\_\_\_ on  $x_0 < x < x_1$  the function y(x) is concave up and estimates of  $y(x_1)$  using Euler's Method will be \_\_\_\_\_. When y'' is \_\_\_\_\_\_ on  $x_0 < x < x_1$  the function y(x) is con-

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