Differential Equations

Math 341 Spring 2005 MWF 8:30 - 9:25am Fowler North 2

`http://faculty.oxy.edu/ron/math/341`

Class 25: **Friday April 1**

TITLE *The Inverse Laplace Transform*
CURRENT READING Zill 7.2

Homework Set #10
Zill, Section 7.1: 4*, 9*, 11*, 20* *EXTRA CREDIT 41*
Zill, Section 7.2: 2*, 7*, 11*, 25*, 41* *EXTRA CREDIT 42*

SUMMARY
We will start using the Lapace Transform (and the Inverse Laplace Transform) to solve differential equations.

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1. The Inverse Laplace Transform

DEFINITION: **Inverse Laplace Transform**

If $F(s)$ represents the Laplace Transform of a function $f(t)$ such that $\mathcal{L}[f(t)] = F(s)$ then the Inverse Laplace Transform of $F(s)$ is $f(t)$, i.e. $\mathcal{L}^{-1}[F(s)] = f(t)$.

Exercise

Compute $\mathcal{L}^{-1}[\frac{1}{s^5}]$ and $\mathcal{L}^{-1}[\frac{1}{s^2+7}]$

EXAMPLE Let's show that $\mathcal{L}^{-1}[\frac{-2s+6}{s^2+4}] = -2\cos(2t) + 3\sin(2t)$

2. Transforming A Derivative

We can show that $\mathcal{L}[f'(t)] = sF(s) - f(0)$

RECALL The function e^{At} is a solution of $(e^{At})' = Ae^{At}$ where $e^{A0} = \mathcal{I}$. We can now show that $e^{At} = \mathcal{L}^{-1}[(s\mathcal{I} - A)^{-1}]$.

Inverse Laplace Transforms	
$F(s)$	$f(t) = \mathcal{L}^{-1}[F(s)]$
$\frac{1}{s}$	1
$\frac{n!}{s^{n+1}}$	t^n
$\frac{1}{s-a}$	e^{at}
$\frac{k}{s^2+k^2}$	$\sin(kt)$
$\frac{s}{s^2+k^2}$	$\cos(kt)$
$\frac{k}{s^2-k^2}$	$\sinh(kt)$
$\frac{s}{s^2-k^2}$	$\cosh(kt)$

THEOREM

If $f, f', f'', f^{(n-1)}, \ldots, f^{(n-1)}$ are continuous on $[0, \infty)$ and of exponential order c and if $f^{(n)}$ is piecewise continuous on $[0, \infty)$, then $\mathcal{L}[f^{(n)}(t)] = s^n F(s) - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0)$

3. Using Transforms To Solve Differential Equations

EXAMPLE **Zill, Example 4, page 289.** Use the Laplace Transform to solve the initial value problem $\dfrac{dy}{dt} + 3y = 13\sin 2t, \quad y(0) = 6$

Exercise **Zill, Example 5, page 289.** Use the Laplace Transform to solve the initial value problem $y'' - 3y' + 2y = e^{-4t}, \quad y(0) = 1, \quad y'(0) = 5$