Differential Equations

Math 341 Spring 2005 MWF 8:30 - 9:25am Fowler North 2
 http://faculty.oxy.edu/ron/math/341

Class 7: Wednesday February 2

TITLE *Exact Differentials and Exact Equations*
CURRENT READING Zill, 2.4

Homework Set #3
Zill, Section 2.2: 2*, 3, 4*, 7, 9*, 16, 17*, 23*, 25, 26 *EXTRA CREDIT 31, 39, 44*
Zill, Section 2.3: 3, 4, 7*, 9, 15*, 22*, 29, 30, 31, 34* *EXTRA CREDIT 35, 43, 50*
Zill, Section 2.4: 5, 10*, 11*, 19, 27*, 30, 32, 38* *EXTRA CREDIT 42, 44*

SUMMARY
We will be introduced to another technique for finding the solution of a DE. This time we restrict the class of DEs we are trying to solve to be **exact differential equations**.

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1. Exact Differentials

RECALL The contours of a surface $z = f(x, y)$ are defined by the equation $f(x, y) = c$. An expression for the change in z, $dz = f_x\ dx + f_y\ dy = 0$.

DEFINITION: **exact diferential**
An **exact differential** has the form $M(x, y)\ dx + N(x, y)\ dy$ in a region R of the xy-plane if it corresponds to the differential of a function $f(x, y)$ defined in R. The equation $M(x, y)\ dx + N(x, y)\ dy = 0$ is called an **exact equation** or **exact differential equation** if the expression on left hand side of the equation is an exact differential.

THEOREM
Let $M(x, y)$ and $N(x, y)$ be continuous and have continuous first partial derivatives in a rectangular region $a < x < b, c < y < d$.
$M(x, y)\ dx + N(x, y)\ dy$ is an exact differential IF AND ONLY IF $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

Proof
$M_y = N_x \Rightarrow Mdx + Ndy$ is an exact differential

$Mdx + Ndy$ is an exact differential $\Rightarrow M_y = N_x$

EXAMPLE **Zill, page 69, Example 1.** Show that $2xy\ dx + (x^2 - 1)\ dy = 0$ is an exact differential equation and then solve the exact differential equation.

Exercise **Zill, page 70, Example 3.** Solve $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y(1 - x^2)}, \quad y(0) = 2.$

2. Integrating Factors for Exact Differentials

If $M(x, y)dx + N(x, y)dy = 0$ is NOT an exact DE we can try and make it so by multiplying by an integrating factor $\mu(x, y)$ similar to what we used for linear first-order DEs.

If $(M_y - N_x)/N$ is a function of x only, then $\mu(x, y) = e^{\int \frac{M_y - N_x}{N} dx}$

If $(N_x - M_y)/M$ is a function of y only, then $\mu(x, y) = e^{\int \frac{N_x - M_y}{M} dy}$

EXAMPLE **Zill, page 72, Example 4.** Let's make $xy\ dx + (2x^2 + 3y^2 - 20)\ dy = 0$ an exact DE and write down the solution of the exact DE.