

Quiz 8

Differential Equations

Name: _____

Time Begun: _____

Time Ended: _____

Friday April 1

Ron Buckmire

Topic : Bifurcations in Systems of Differential Equations

The idea behind this quiz is to provide you with an opportunity to think about how bifurcations can occur in linear systems of DEs.

Reality Check:

EXPECTED SCORE : _____/10

ACTUAL SCORE : _____/10

Instructions:

0. Please look for a hint on the course website at <http://faculty.oxy.edu/ron/math/341/> in the News section.
1. Once you open the quiz, you have as much time as you like to complete it, but please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. **You must work alone.**
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday April 4**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the linear system of ordinary differential equations with a parameter a

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} -2 & a \\ -2 & 0 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Let's describe how the phase portrait changes as a varies from $-\infty$ to $+\infty$.

(a) *3 points.* Compute the general solution and sketch the phase portrait when $a = -3/2$. Describe the stationary point at the origin.

(b) *3 points.* Compute the general solution and sketch the phase portrait when $a = 1$. Describe the stationary point at the origin.

(c) *4 points.* For what value(s) of a does the system change its nature (i.e. bifurcate) and what does the phase portrait look like then?