Quiz **2** — Differential Equations

Name: ______________________________

Friday January 28
Ron Buckmire

Time Begun: ____________________
Time Ended: ____________________

Topic : Analyzing The Logistic Equation

The idea behind this quiz is to provide you with an opportunity to illustrate your understanding of first-order ordinary differential equations and their solutions

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `blackboard.oxy.edu`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday January 31**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the autonomous differential equation $\dfrac{dP}{dt} = kP(M - P)$
where $P(t)$ is the population in an environment which can only sustain M individuals and k is the constant of proportionality.

(a) *4 points.* Find and classify all the equilibrium points of this differential equation and draw the phase line.

(b) *3 points.* Show that $\dfrac{d^2P}{dt^2} = 2k^2P(P - \dfrac{M}{2})(P - M)$ and that $\begin{cases} P'' > 0, & \text{when } 0 < P < \frac{1}{2}M \\ P'' < 0, & \text{when } \frac{1}{2}M < P < M \\ P'' > 0, & \text{when } P > M \end{cases}$

(c) *3 points.* Use information from part (a) and part (b) to **carefully** sketch solution curves which go through (i) $P(0) = 2M$, (ii) $P(0) = M/2$ and (iii) $P(0) = M/4$.