

BONUS QUIZ 3

Differential Equations

Name: _____

Friday February 18
Ron Buckmire

Topic : Abel's Formula for the Wronskian

The idea behind this bonus quiz is to provide you with an opportunity to discover an interesting result involving the Wronskian.

Reality Check:

EXPECTED SCORE : _____/10

ACTUAL SCORE : _____/10

Instructions:

1. Please look for a hint on this quiz posted in the News section of <http://faculty.oxy.edu/ron/math/341>
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Wednesday February 23**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Recall that the homogeneous, linear, second-order differential equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ possesses two linearly independent solutions $y_1(x)$ and $y_2(x)$ when the Wronskian $W(y_1(x), y_2(x))$, defined as $W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ is not equal to zero.

(a) 3 points. Show that the derivative of the Wronskian can be written as $W'(x) = \begin{vmatrix} y_1 & y_2 \\ y_1'' & y_2'' \end{vmatrix}$.

(b) 3 points. By using the fact that y_1 and y_2 are solutions to the DE and recalling your knowledge about determinants from *Linear Systems* show that you can re-write $W'(x) = -P(x)W(x)$.

(c) 2 points. Solve the differential equation for $W(x)$ in terms of $P(x)$ to obtain **Abel's Formula** for the Wronskian.

(d) 2 points. Given that $y(x) = c_1e^{2x} + c_2e^{-x}$ is the fundamental solution to $y'' - y' - 2y = 0$, use the standard definition of the Wronskian to confirm Abel's formula for the Wronskian.