Differential Equations

Math 341 Spring 2005 — MWF 8:30 - 9:25am Fowler North 2

 — http://faculty.oxy.edu/ron/math/341

Report on Exam 2

Point Distribution (N=23)

Range	100+	90+	85+	80+	75+	70+	65+	60+	55+	50+	45+	40+	40-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	2	3	0	2	2	2	0	1	3	0	4	0	4

Comments

Summary Overall class performance was mixed. Almost half the class (11 of 23) earned a 90 or above. The average score on the exam was a 65 with a standard deviation of 22. The highest score was a 105 and the low score was a 30.

#1 This problem is about what to do when you have repeated eigenvalues and thus you don't have enough eigenvectors. The answer is, you use Generalized Eigenvectors. **(a)** The eigenvalues of $A = \begin{bmatrix} 12 & -9 \\ 4 & 0 \end{bmatrix}$ are 6 and 6 with one corresponding eigenvector $\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. The generalized eigenvector $\vec{y}$ which solves the equations $(A - 6I)\vec{y} = \vec{x}$. This means that $2y_1 - 3y_2 = 1$ where $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. You can pick any values y_1 and y_2 which satisfy this equation. **(b)** The general solution is $\vec{f}(t) = c_1\vec{x}e^{6t} + c_2e^{6t}(\vec{x}t + \vec{y})$. **(c)** One can solve for c_1 andc_2 when $t = 0$ and $\vec{f}(0) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$.

#2 **(a)** $ty'' + y' + ty = 0$ is really just a form of Bessel's Equation of order 0, i.e. $t^2y'' + ty' + (t^2 - 0^2)y = 0$ so $y(t) = c_0J_0(t) + c_1Y_0(t)$. Using the initial conditions one can see that $c_1 = 0$ and $c_0 = 1$. **(b)** This is a very cool result which allows one to apply Laplace Transforms to solve a broader class of linear ODEs. **(c)** By using separation of variables and assuming $Y(0) = A$ produces the result that $Y(S) = A(s^2+1)^{-1/2}$ **(d)** Since $\lim_{s\to\infty} sY(s) = y(0)$ one needs to show that $\lim_{s\to\infty} As(s^2+1)^{-1/2} = A = y(0) = 1$ so this shows that $Y(s) = \dfrac{1}{\sqrt{s^2+1}} = \mathcal{L}[J_0(t)]$.

#3 **(a)**. You can show that the $x = 0$ is a regular singular point by computing $\lim_{x\to 0} xP(x) = p_0$ and $\lim_{x\to 0} x^2Q(x) = q_0$ where the DE is $y'' + P(x)y' + Q(x)y = 0$. Since $p_0 = 1$ and $q_0 = 0$ the indicial equation is simply $r^2 = 0$. **(c)** You can use Frobenius Theorem to obtain a series solution of the form $y = \sum_{n=0}^{\infty} a_nx^n$ of the Laguerre's equation $xy'' + (1 - x)y' + \lambda y = 0$ and obtain the given recurrence relation that $a_n = \dfrac{n-1-\lambda}{n^2}a_{n-1}$ **(d)** Using the result from **(c)** $y(x) = a_0 + a_0\dfrac{(-\lambda)}{1^2}x + \dfrac{(-\lambda)(1-\lambda)}{1^2\cdot 2^2}x^2 + a_0\dfrac{(-\lambda)(1-\lambda)(2-\lambda)}{1^2\cdot 2^2\cdot 3^2}x^3 + a_0\dfrac{(-\lambda)(1-\lambda)(2-\lambda)(3-\lambda)}{(4!)^2}x^4 + \ldots + a_0\dfrac{(-\lambda)(1-\lambda)(2-\lambda)\ldots(n-\lambda)}{(n!)^2}x^{n+1}$ when $\lambda = n$ one obtains a different n^{th} degree polynomial known as a Laguerre Polynomial for every positive integer value of λ.