Differential Equations

Math 341 Spring 2005 MWF 8:30 - 9:25am Fowler North 2

 http://faculty.oxy.edu/ron/math/341

Report on Exam 1

Point Distribution (N=23)

Range	100+	93+	90+	87+	83+	80+	77+	73+	70+	65+	60+	50+	50-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	5	3	3	1	1	0	1	1	1	1	3	1	2

Comments

Summary Overall class performance was surprisingly bimodal. Almost half the class (11 of 23) earned a 90 or above. The average score on the exam was a 80 with a standard deviation of 20. The highest score was a 109.

#1 This problem is about highlighting that the existence and uniqueness theorem for $y' = f(x,y), y(a) = b$ only insures existence of a unique solution in a small neighborhood of the given initial condition (a,b). **(a)** Since $y' = 2xy^2$ then $f(x,y) = 2xy^2$ and $f_y(x,y) = 4xy$ which exist and are continuous for every point in the (x,y)-plane. Since the given initial condition is $y(0) = 1/c^2$ with $c > 0$, the existence and uniqueness theorem guarantees a unique solution in a region near $(0, 1/c^2)$ for every value of $c > 0$. **(b)** You can obtain the functional form of the solution of the IVP by either using separation of variables or you can just verify that the given one-parameter family $y = 1/(c^2 - x^2)$ satisfies *both* the differential equation and the initial condition. **(c)** This question is geared towards a visual learning style. The point is for you to see that (i) the family of curves is undefined at $x = \pm c$ and (ii) the width of the interval the curve is defined which contains $(0, 1/c^2)$ varies with the value of c (and can both get very small and very big). **(d)** The interval of definition of the solution of the IVP must contain $(0, 1/c^2)$ and it must be a contiguous interval, so it is $-c < x < c$. This is different from the interval the family of curves is defined on, which is $\{-c > x\} \cup \{-c < x < c\} \cup \{x > c\}$.

#2 This problem is oriented towards visual learning styles but also verbal/analytic learning styles since you have to explain your choice. From the lack of any lineal elements along $x = -1$ and $x = 1$ you know that the derivative is undefined at those values, so that tells you it could be 2, 4 or 5. But 5 would also be undefined at $x = 0$. At $x = 0$ clearly the slope is always positive when $y > 0$, so this lets you know the correct choice is 4. The solution curve through $(0, 1/2)$ is trapped between $-1 < x < 1$. (Recall Question 1(c)). **(c)**Even though the DE #4 is not autonomous we can still tell it has a constant, equilibrium solution, namely $y = 0$. Because the DE is not autonomous this solution has conditional stability, in that its stability depends on x. As you can tell from the graph, when $|x| < 1$ solutions move away from the $y = 0$ line so it is UNSTABLE, and when $x < -1$ or $x > 1$ solutions move towards $y = 0$ so it is STABLE then. You earned full credit if you noticed that $y = 0$ is an equilibrium solution. $x = 1$ and $x = -1$ are not solutions! These vertical lines are not functions, so they can not be solutions of the DE.

#3 TRUE/FALSE questions are not by definition easy! Whenever you are taking a test you should be executing an optimization algorithm to maximize your points. In TRUE/FALSE questions this means that what you write to support your determination of TRUE or FALSE needs to be correct and indicate how much you know about the material. For a statement to be TRUE it must always be TRUE, for a statement to be FALSE all you need to do is think of one example which makes the statement false. **(a)**. You can show that one-parameter family of curves $y(x) = \frac{1}{2}(3 - Ce^{-x^2})$ solves the DE $y' + 2xy = 3x$ but since you are given a specific initial condition $y(0) = 1$ the solution picks out a particular value of C (equal to 1). This is not a family of curves, it is a particular curve. Therefore, the given statement is **FALSE.** **(b)** First of all you want to recognize that $x^2y'' = 2y$ is actually a Cauchy-Euler equation, $x^2y'' - 2y = 0$ where $a = 1, b = 0, c = -2$. Obtaining the auxiliary equation gives you $m^2 - m - 2 = 0$ which factors to be $(m+1)(m-2) = 0$ and has solution $m = -1$ and $m = 2$. Thus the solutions $y_1 = x^2$ and $y_2 = x^{-1}$ are correct, but are they linearly independent? Check the Wronskian: is it never zero? Yes! (It's a constant.) Therefore, the given statement is **TRUE**. Both of these questions were geared towards symbolic/calculation and verbal/analytic learning styles.

#4 Undetermined Coefficients, Mathematical Models. The given equation is $y^{(4)} - k^2y'' = q(x)$. **(a)** When $q(x) = 0$ the auxiliary equation is $m^4 - k^2m^2 = 0$ which has solutions $m = 0, 0, k, -k$, so the homogeneous solution has the form $y = c_1 + c_2x + c_3e^{kx} + c_4e^{-kx}$. **(b)** If $q(x) = 1$ then you have to find the particular solution which one would guess has the form $y_p = A$ but since that is already duplicated in the homogeneous solution the correct form of the $y_p = Ax^2$. When you plug this form into the nonhomogeneous equation leads to the result that $A = \frac{-1}{2k^2}$. So the general solution is $y = c_1 + c_2x + c_3e^{kx} + c_4e^{-kx} - \frac{x^2}{2k^2}$ **(c)** Even though the exact form of the general solution looks intimidating, one can immediately check your answers to part (a) from it. By setting $q(x) = 1$ the integrals are very easily evaluated, thus leading to the same answer you got in (b). Notice that any constant of integration that might show up is already present in the homogeneous solution. **(d)**. There's actually a typo in the given general solution, so that the last term should be $-e^{-kx}/k^3$ not $+e^{-kx}/k^3$. So, if you attempted the BONUS question at all, I gave you the full 10 points. This question is geared towards calculation/symbolic learning styles.