Math 340 Fall 2015

Quiz **11** Ordinary Differential Equations

Name: _____ **Prof. Ron Buckmire**

ASSIGNED: Friday November 20

Time Begun: _____
Time Ended: _____ **DUE: Monday November 23**

Topic : Advanced Laplace Transforms

The idea behind this quiz is to provide you with an opportunity to demonstrate your ability with inverting Laplace Transforms of a complicated function.

Reality Check:

EXPECTED SCORE : _____**/10** ACTUAL SCORE : _____**/10**

Instructions:

0. Please look for a hint on this quiz posted to `http://sites.oxy.edu/ron/math/340/15/`
1. Once you open the quiz, you have **30 minutes** to complete it, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. **You must work alone**.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution. Use complete sentences wherever possible.
6. Relax and enjoy...
7. **This quiz is due at the beginning of class on Monday November 23**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 340 Fall 2015 **SHOW ALL YOUR WORK** Quiz **ELEVEN**

1. The goal of this problem is to find the function $f(t)$ whose Laplace Transform is

$$F(s) = A(s) - B(s) = \frac{1}{s^2} - \frac{e^{-s}}{s(1-e^{-s})}, \quad s > 0$$

(a) *2 points.* Compute $\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = a(t)$.

(b) *2 points.* If one considers $\frac{1}{1-e^{-s}}$ as the sum of a geometric series $\sum_{k=0}^{\infty} ar^k$ with first term $a = 1$ and ratio $r = e^{-s}$ then show that $\frac{e^{-s}}{s(1-e^{-s})}$ can be written as $\sum_{k=1}^{\infty} \frac{e^{-ks}}{s} = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \dots$

(c) *3 points.* Recall that $\mathcal{L}^{-1}\left[e^{-as}F(s)\right] = f(t-a)\mathcal{H}(t-a)$. Using the result given in (b), compute $\mathcal{L}^{-1}\left[\frac{e^{-s}}{s(1-e^{-s})}\right] = b(t)$.

(d) *3 points.* Give a sketch of $a(t)$, $b(t)$ and $f(t) = a(t) - b(t)$ below for $t > 0$ (Use different pairs of axes for each graph.)