Math 340 Fall 2015

BONUS QUIZ 2 — Ordinary Differential Equations

Name: ________________________________ **Prof. Ron Buckmire**

Assigned: Friday October 23

Time Begun: ______________________

Time Ended: ______________________ **DUE: Monday October 26**

Topic : Visualizing Solutions of Linear Systems of ODEs

The idea behind this quiz is to provide you with an opportunity to illustrate your understanding of solution techniques for systems of n linear ordinary differential equations.

Reality Check:

EXPECTED SCORE : __________**/5** ACTUAL SCORE : __________**/5**

Instructions:

0. BEFORE you open the quiz, look for a hint at `sites.oxy.edu/ron/math/340/15`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This bonus quiz is due on Monday October 26**, at the beginning of class.

Pledge: I, ______________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the system of ordinary differential equations

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

(a) *1 point.* Show that the matrix A has eigenvalues 0 and -1 and eigenvectors which are multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Write down the 2-parameter general solution of the system $\frac{d\vec{x}}{dt} = A\vec{x}$.

(b) *2 points.* Find the exact solution $\vec{x}(t)$ for each of the trajectories which go through the points **A**$(1, 1)$, **B** $(0, -2)$ and **C** $(4, 0)$ at $t = 0$.

(c) *2 points.* On the figure below clearly indicate the trajectories for each of the solutions which start at **A**$(1, 1)$, **B** $(0, -2)$ and **C** $(4, 0)$ ends up as $t \to \infty$. Label these endpoints $\mathbf{A}'$, $\mathbf{B}'$ and $\mathbf{C}'$ respectively.