Math 340 Fall 2015

BONUS QUIZ 1 — Ordinary Differential Equations

Name: ____________________ **Prof. Ron Buckmire**

Assigned: Friday September 4

Time Begun: ____________________
Time Ended: ____________________

DUE: Wednesday September 9

Topic : Analyzing a Clairault Equation

The idea behind this bonus quiz is to provide you with an opportunity to illustrate your understanding of singular solutions to ordinary differential equations.

Reality Check:

EXPECTED SCORE : __________**/5** ACTUAL SCORE : __________**/5**

Instructions:

0. BEFORE you open the quiz, look for a hint at `sites.oxy.edu/ron/math/340/15`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This bonus quiz is due on Wednesday September 9**, at the beginning of class.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the first-order, nonlinear, Clairault ordinary differential equation

$$y = x\left(\frac{dy}{dx}\right) - \frac{1}{4}\left(\frac{dy}{dx}\right)^2$$

(a) *1 point.* Confirm that the 1-parameter family of solutions to the given ODE is the set of **lines** of the form $y = Cx - \frac{1}{4}C^2$.

(b) *3 points.* Show that the lines $y = Cx - \frac{1}{4}C^2$ are tangent to the curve $y = x^2$ at the point $\left(\frac{C}{2}, \frac{C^2}{4}\right)$ and sketch the curve and its tangents below for at least 4 values of C.

(c) *1 point.* Explain how parts (a) and (b) imply that $y = x^2$ is a singular solution of the given Clairault ODE and confirm this result. [HINT: A singular solution of an ODE is one which solves the ODE but is not a member of the family of solutions.]