Math 312 Spring 98

Quiz 7

Complex Analysis

Name: ____________________________

Date: ____________________________
Time Begun: ____________________________
Time Ended: ____________________________

Friday March 27
Ron Buckmire

Topic : Cauchy's Integral Formula and its implications

The idea behind this quiz is for you to gain more experience with the applications of Cauchy's Integral Formula and evaluating contour integrals in general.

Instructions:

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Sometime over the weekend I will post a hint on solving this quiz on the Complex Analysis wwwboard at `http://abacus.oxy.edu/wwwboard/complex` . You can access the board by using the login and password `complex`. If you do not understand the hint or have any other questions you should post a response on the wwwboard.
7. Relax and enjoy...
8. **This quiz is due on Monday March 30**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. (*5 points*) One interesting application of contour integration is the ability to find the area of odd shapes in the plane. If we denote the area enclosed by a positively-oriented contour C by $\boldsymbol{A}$, then

$$A = \frac{1}{2i}\oint_C \overline{z}\,dz$$

Recalling that the parametrization given by $z(t) = a\cos t + ib\sin t, 0 \le t \le 2\pi$ represents an elliptical contour C with horizontal axis a and vertical axis b use the formula for $\boldsymbol{A}$ to compute the area enclosed by an ellipse. (Your final answer should only involve π, a and b.)

2. (*5 points*) Evaluate $\displaystyle\oint \frac{dz}{e^{\pi iz}(2z-1)}$ around $\frac{x^2}{9}+\frac{y^2}{4}=1$ in a counter-clockwise direction.

BONUS: (*5 points*) Evaluate $\displaystyle\oint \frac{dz}{e^{\pi iz}(2z-1)^3}$ around $\frac{x^2}{9}+\frac{y^2}{4}=1$ in a counter-clockwise direction.