Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am
Buckmire — Fowler 112

Homework Set 2

ASSIGNED: Fri Feb 6 1998 — **DUE: Fri Feb 13 1998**

1. Write each of the following functions in the form $f(z) = u(x,y) + iv(x,y)$

   (a) $f(z) = z^3 + iz + 1$

   (b) $f(z) = \frac{1}{z}$

   (c) $\frac{z+i}{z^2+1}$

2. Compute $f'(z)$ for each function in the previous question

3. Consider $f(z) = \frac{2z+1}{3z-2}$. Write down

   (a) The domain of definition of f

   (b) the range of f

   (c) $f(f(z))$

   (d) $f\left(\frac{1}{z}\right)$

   (e) $\lim_{z\to 0} f(z)$

   (f) $\lim_{z\to\infty} f(z)$

4. Prove that $f(z) = \overline{z}$ is continuous everywhere in the complex plane.

5. Given $F(z) = z + i, G(z) = e^{\frac{i\pi}{4}}, H(z) = z/2$ sketch the image (and describe it in terms of complex inequalitites) of applying the following composite mappings on the semi-disk $|z| \leq 2 \cap \text{ Im } z \geq 0$

   (a) $F(z)$

   (b) $G(z)$

   (c) $H(z)$

   (d) $F(G(z))$

   (e) $G(H(z))$

   (f) $H(F(z))$

   (g) $F(G(H(z)))$

6. Prove that Re z, Im z and $|z|^2$ are nowhere differentiable.

7. The mapping $w = g(z) = 1/z$ is called an **inversion mapping**.

   (a) Show that the circle $|z| = r$ gets mapped onto the circle $|w| = 1/r$

   (b) Show that the ray Arg $z = \alpha, -\pi < \alpha \leq \pi$, gets mapped onto the ray $Argw = -\alpha$

8. Find the range of each of the following functions:

   (a) $g(z) = z^2$ for Re $z \geq 0 \cap$ Im $z \geq 0$

   (b) $h(z) = z + 5$ for Re $z > 0$

9. Given the complex potential $\Phi(\boldsymbol{z}) = \cos\boldsymbol{\alpha} - \boldsymbol{i}\sin\boldsymbol{\alpha z}$

   (a) $Sketchtheequipotentialsandgiveequationsforthem$

   (b) $Sketchthestreamlinesandgiveequationsforthem$

   (c) Find the components V_x and V_y of the velocity vector $\vec{v}$

   (d) What angle does the vector $\vec{v}$ makes with the x-axis?

   (a) $f(z) = z$ Im z

   (b) $f(z) = x^3 + i(1 - y)^3$

10. Determine for what values of z the derivative $f'(z)$ exists:

11. Show that if $f(z)$ is harmonic then $f''(z) = u_{xx} + iv_{xx} = -u_{yy} - iv_{yy}$

12. Find the most general form of $\phi(x, y) = ax^2 + bxy + cy^2$ which is harmonic

NOTES

All homework sets will be due in class *at least* one week from the class they are given out in. You are strongly encouraged to work together on the homework and to come see me for help with any of the problems. Please indicate on your answer sheets the names of the other students with whom you collaborated.