MATH 312
HW 8

4

5.2 : 2, 7, 10, 21, 22, 29*

2. $\int_C 2\bar{z} - z\,dz = \int_0^2 2x - x + i(-2y - y)\,dz = \int_0^2 -t + i(-3t^2-6)(-1+2it)\,dt$

$C: x = -t,\ y = t^2+2,\ 0 \le t \le 2$

$dz = (-1 + i2t)\,dt$

$\int_0^2 t - 2it^2 + i(3t^2+6) + 6t^2 + 12\,dt$

$= \int_0^2 (6t^2 + t + 12)\,dt + i\,(3t^2 + 6 - 2t^2)\,dt$

$= \left(2t^3 + \frac{t^2}{2} + 12t\right)\Big|_0^2 + i\left(\frac{t^3}{3} + 6t\right)\Big|_0^2 = (16 + 2 + 24) + i\,\frac{44}{3} = 42 + i\,\frac{44}{3}$

7. $\int \mathrm{Re}(z)\,dz = \int_0^{2\pi} \cos t \cdot ie^{it}\,dt = \int_0^{2\pi} -\sin t\cos t + i\cos^2 t\,dt$

$C: |z| = 1$

$z(t) = e^{it},\ 0 \le t \le 2\pi \qquad z' = ie^{it}\,dt$

$= -\int_0^{2\pi} \frac{\sin 2t}{2}\,dt + i\int_0^{2\pi} \frac{\cos 2t + 1}{2}\,dt$

$= -\frac{\cos 2t}{4}\Big|_0^{2\pi} + i\left(\frac{\sin 2t}{4} + \frac{t}{2}\right)\Big|_0^{2\pi} = \boxed{\pi i}$

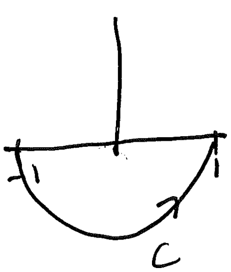
10. $\int_C (x^2 - iy^3)\,dz = \int_{-\pi}^{0} (\cos^2 t - i\sin^3 t)(i\cos t \cancel{+ i\sin t})\,dt$

$z = e^{it},\ -\pi \le t \le 0 \quad = \int_{-\pi}^{0} -\cos^2 t\sin t + \sin^3 t\cos t\,dt$

$z' = ie^{it} \qquad + i\int_{-\pi}^{0} \sin^4 t + \cos^3 t\,dt$

$x = \cos t$

$y = \sin t$



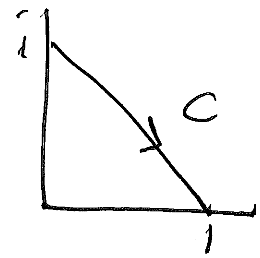
$= \frac{2}{3} + \frac{3\pi}{8}i$ (Wolfram Alpha ☺)

MATH 312

HW 8

$\boxed{5}$

21 $\displaystyle\int_C z^2 - z + 2\,dz$



$z(t) = t + (1-t)i \,, \quad 0<t<1$

$z'(t) = 1 - i$

$$\int_0^1 (1-i)\{[t^2-(1-t)^2+2t(1-t)i] - [t+(1-t)i] + 2\}\,dt$$

$$\int_0^1 (1-i)\{\cancel{t^2} - (1 + \cancel{t^2} - \cancel{2t}) + \cancel{2t}i - 2t^2 i - \cancel{t} - i + \cancel{ti} + \cancel{2}\}\,dt$$

$$(1-i)\int_0^1 \{-1 + \cancel{2t} + 3ti \overset{-2t^2}{+ \cancel{2}} - \cancel{t} - i\}\,dt = (1-i)\int_0^1 1 + t + i(3t-1 \underset{-2t^2}{)}\,dt$$

$$= (1-i)\left\{\left(t+\frac{t^2}{2}\right)\Big|_0^1 + i\left(\frac{3t^2}{2}-\frac{2t^3}{3}-t\right)\Big|_0^1\right\} = (1-i)\left(\frac{3}{2} + i\left\{\frac{3}{2}-\frac{2}{3}-1\right\}\right)$$

$$= \cancel{(1-i)\left\{\left(\frac{3}{2}+i\frac{1}{2}\right)-0-0\right\}} \qquad = (1-i)\left(\frac{3}{2} + -\frac{i}{6}\right)$$

$$= \cancel{\frac{3}{2}+\frac{i}{2}-\frac{3i}{2}+\frac{1}{2}} \qquad = \frac{3}{2} - \frac{3i}{2} - \frac{i}{6} + \frac{1}{6}$$

$$= \frac{8}{6} - \frac{10i}{6} = \frac{4}{3} - \frac{5}{3}i$$

$$\int_1^i z^2 - z + 2\,dz = \left(\frac{z^3}{3} - \frac{z^2}{2} + 2z\right)\Big|_1^i (-1)$$

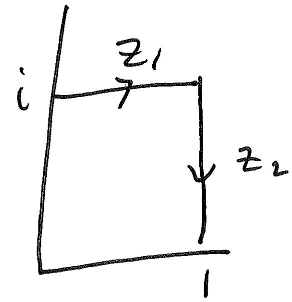
$$= \left[\left(\frac{i^3}{3} - \frac{i^2}{2} + 2i\right) - \left(\frac{1}{3} - \frac{1}{2} + 2\right)\right](-1)$$

$$= \left[\left(-\frac{i}{3} + \frac{1}{2} + 2i\right) - \left(\frac{11}{6}\right)\right](-1) = \frac{5i}{3} - \frac{8}{6} = \left(\frac{5i}{3} - \frac{4}{3}\right)(-1)$$

$$= \frac{4}{3} - \frac{5i}{3}$$

HW8 MATH 312 $\qquad$ L6.

22. $\int z^2 - z + 2\,dz$



$\cancel{z(t) = (1+i)(t-1)i}$

$\cancel{= t + it -}$

$\int_0^1 (t+i)^2 - (t+i) + 2\,dt \qquad z_1' = 1 \qquad z_1(t) = i(1-t) + (1+i)(t)$

$= i - ti + t + it$

$= i + t,\ 0<t<1$

$+ \int_0^1 -i[[1+i(1-t)]^2 - [1+i(1-t)] + 2]\,dt \qquad z_2' = -i \qquad z_2(t) = (1+i)(1-t) + 1\cdot t$

$= 1 + i - t - it + t$

$= 1 + i - it$

$= 1 + i(1-t),\ 0<t<1$

$= \int_0^1 t^2 - 1 + 2it - t - i + 2\,dt$

$+ \int_0^1 -\cancel{2}i + i - \cancel{1} + \cancel{t} - i + i(1-t)^2 + 2(1-t)\,dt$

$= \int_0^1 t^2 + 1 - t + i(2t-1)\,dt + \int_0^1 + 1 + 1t + i(-2+1-2t+t^2)\,dt$

$= \left(\frac{t^3}{3} + t - \frac{t^2}{2}\right)\Big|_0^1 + i(t^2 - t)\Big|_0^1 + \left(1t + \frac{1t^2}{2}\right)\Big|_0^1 + i\left(-t - t^2 + \frac{t^3}{3}\right)\Big|_0^1$

$= \left(\frac{1}{3} + 1 - \frac{1}{2}\right) - 0 + i(1-1) + i\,0 + \left(1 + \frac{1}{2}\right) - (0) + i\left(-1 - 1 + \frac{1}{3}\right)$

$= \cancel{\frac{5}{6} - \frac{3}{2} + i\left(-\frac{4}{3}\right)} \qquad \frac{4}{3} - \frac{5}{3}i$

$= \cancel{-\frac{4}{6} + i\left(\frac{4}{3}\right)}$

MATH312
HW8

29*

(a) $\int_C dz = \int_{z_0}^{z_n} dz = 1\Big|_{z_0}^{z_n} = z_n - z_0$

(b) $\int_C dz = \int_{2i}^{-2i} dz = 1\Big|_{2i}^{-2i} = -2i - 2i = -4i$

(c) $\oint_C dz = 0$