HW 7
Math 312

Sec 4.5: 9*, 10*

Chap 4 Review: 1, 2, 3, 9, 12, 14, 25, 26

1. "$|e^z| = 1$, then z is a pure imaginary number."

TRUE $|e^z| = |e^{x+iy}| = e^x = 1 \Leftrightarrow x = 0$ so $z = iy$.

2. "$Re(e^z) = \cos(y)$"

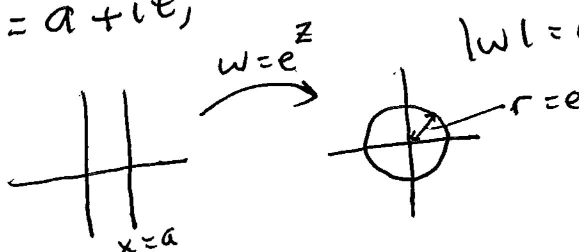
FALSE $Re(e^z) = Re[e^x(\cos y + i\sin y)] = e^x \cos y$

3. "The mapping $w = e^z$ takes vertical lines onto horizontal lines in the w-plane."

FALSE

$z = a + it, \ -\infty < t < \infty$ $\quad w = e^{a+it} = e^a e^{it}, \ -\infty < t < \infty$

$|w| = e^a \leftarrow$ circles of radius e^a



9. "$Ln\left(\frac{1}{z}\right) = -Ln(z)$ for all non zero z."

TRUE $Ln\left(\frac{1}{z}\right) = Ln\,1 - Ln(z) = 0 - Ln(z)$

12. "The principal value of i^{i+1} is $e^{-\frac{\pi}{2} + i}$."

FALSE $i^{i+1} = e^{(i+1)Ln(i)} = e^{(i+1)(0 + i\frac{\pi}{2})} = e^{(i+1)i\frac{\pi}{2}} = e^{-\frac{\pi}{2} + i\frac{\pi}{2}}$

14. "$\cos z$ is a periodic function with a period of 2π."

TRUE $\cos(z + 2\pi) = \dfrac{e^{i(z+2\pi)} + e^{-i(z+2\pi)}}{2}$

$= \dfrac{e^{iz} \cdot e^{2\pi i} + e^{-iz} \cdot e^{2\pi i}}{2} = \dfrac{e^{iz} + e^{-iz}}{2}$

$= \cos(z)$

MATH 312
HW 7

25 $e^{iz} = 2$, then $z = -i\ln(2)$

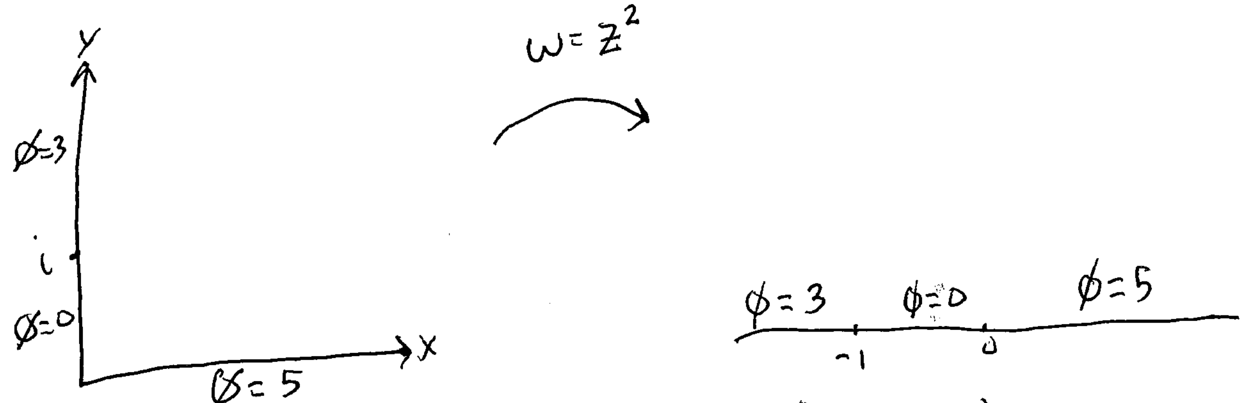
~~27~~ $z = -\ln 2$

26. $\mathrm{Ln}(e^{1-\pi i}) = \mathrm{Ln}(e^{1} \cdot e^{-\pi i}) = \mathrm{Ln}(-e)$

$= \log_e(|-e|) + i\,\mathrm{Arg}(-e)$

$= 1 - \pi i$

4.5 9*, 10*



$$\phi(x,y) = K_n + \frac{1}{\pi}\sum_{j=1}^{n}(K_{j-1}-K_j)\,\mathrm{Arg}(z-x_j)$$

$K_0 = 3$, $x_0 = -1$

$K_1 = 0$, $x_1 = 0$

$K_2 = 5$

$$\phi(x,0) = \begin{cases} 3, & -\infty < u < -1 \\ 0, & -1 < u < 0 \\ 5, & 0 < u < \infty \end{cases}$$

$$\phi(u,v) = 5 + \frac{1}{\pi}(0-5)\,\mathrm{Arg}(w) + \frac{1}{\pi}(3-0)\,\mathrm{Arg}(w+1)$$

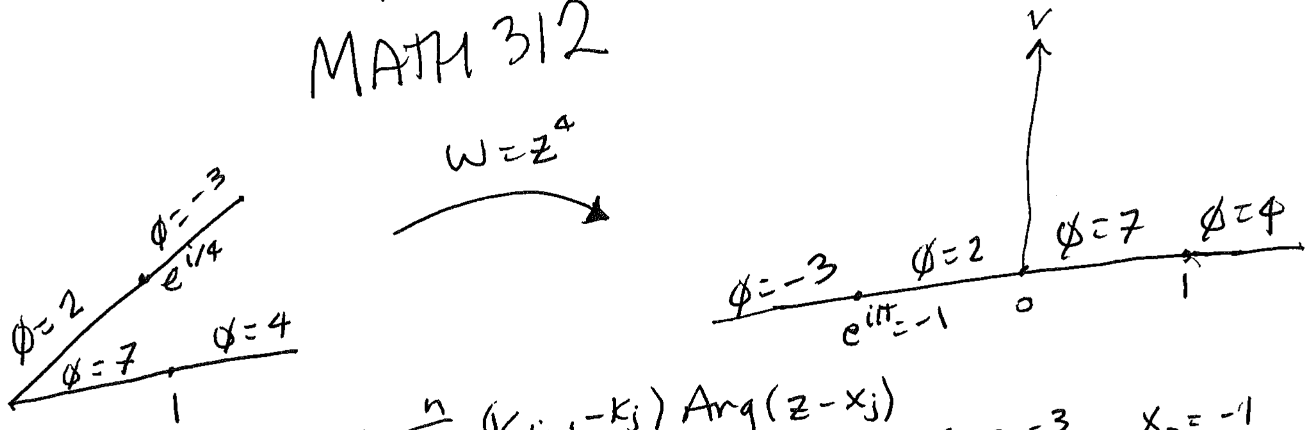
$$= 5 - \frac{5}{\pi}\mathrm{Arg}(w) + \frac{3}{\pi}\mathrm{Arg}(w+1)$$

$$= 5 - \frac{5}{\pi}\mathrm{Arg}(z^2) + \frac{3}{\pi}\mathrm{Arg}(z^2+1)$$

$$\Phi(z) = 5 - \frac{5}{\pi}\mathrm{Ln}(z^2) + \frac{3}{\pi}\mathrm{Ln}(z^2+1)$$

HW7
MATH 312

10.



$$\phi(x,y) = K_n + \frac{1}{\pi}\sum_{j=1}^{n}(K_{j-1}-K_j)\,\mathrm{Arg}(z-x_j)$$

$K_0 = -3$, $K_1 = 2$, $K_2 = 7$, $K_3 = 4$

$x_0 = -1$, $x_1 = 0$, $x_2 = 1$

$$\phi(x,0) = \begin{cases} -3, & -\infty < u < -1 \\ 2, & -1 < u < 0 \\ 7, & 0 < u < 1 \\ 4, & 1 < u < \infty \end{cases}$$

$$\phi(u,v) = 4 + \frac{1}{\pi}(7-4)\,\mathrm{Arg}(w-1) + \frac{1}{\pi}(2-7)\,\mathrm{Arg}(w) + \frac{1}{\pi}(-3-2)\,\mathrm{Arg}(w+1)$$

$$= 4 + \frac{3}{\pi}\mathrm{Arg}(w-1) + \frac{5}{\pi}\mathrm{Arg}\,w - \frac{5}{\pi}\mathrm{Arg}(w+1)$$

$$= 4 + \frac{3}{\pi}\mathrm{Arg}(w-1) + \frac{5}{\pi}\mathrm{Arg}(z^4) - \frac{5}{\pi}\mathrm{Arg}(z^4+1)$$

$$= 4 + \frac{3}{\pi}\mathrm{Arg}(z^4-1) + \frac{5}{\pi}\mathrm{Arg}(z^4) - \frac{5}{\pi}\mathrm{Arg}(z^4+1)$$

$$\Phi(z) = 4 + \frac{3}{\pi}\mathrm{Ln}(z^4-1) - \frac{5}{\pi}\mathrm{Ln}(z^4) - \frac{5}{\pi}\mathrm{Ln}(z^4+1)$$