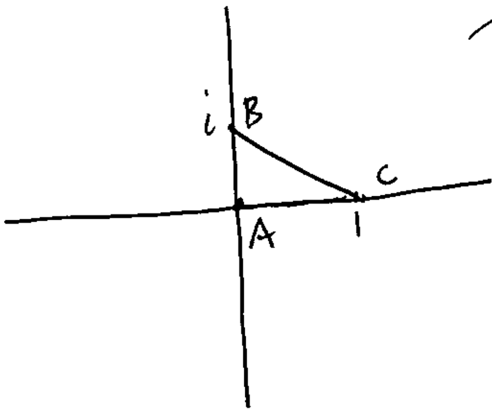
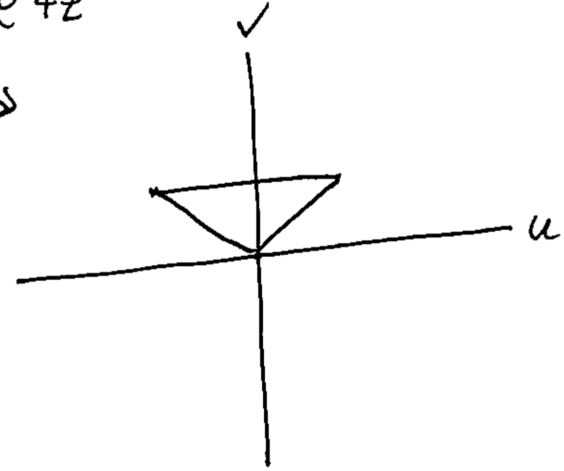
# HW Set 3



2.3
#9

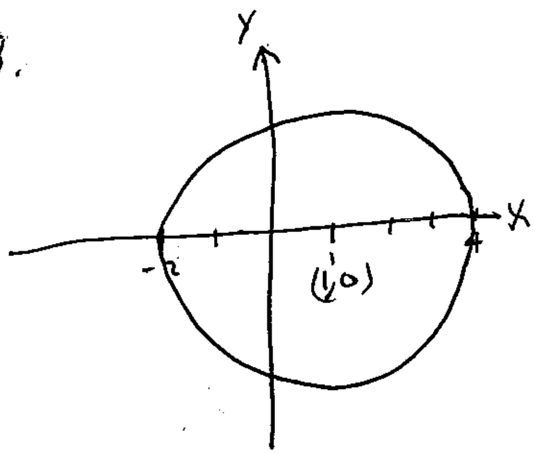
$w = f(z) = e^{\frac{i\pi}{4}z}$

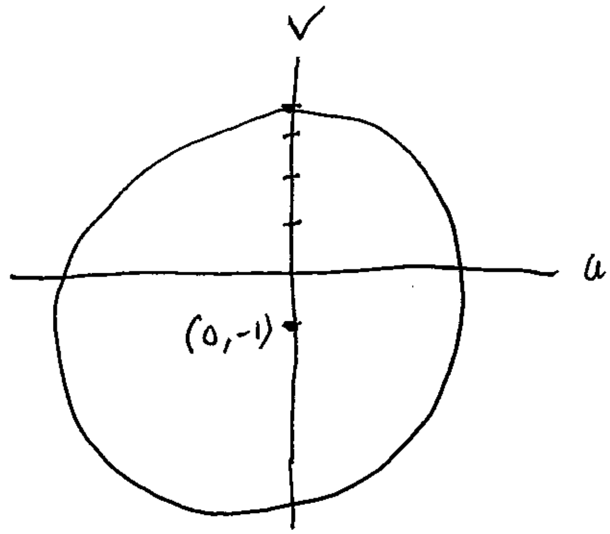



A: $f(0) = e^{i\pi/4} \cdot 0 = 0$
B: $f(1) = e^{i\pi/4} \cdot 1 = e^{i\pi/4}$
C: $f(i) = e^{i\pi/4} \cdot i = e^{i\pi/4} e^{i\pi/2} = e^{3i\pi/4}$

#18.




$S: |z-1| = 3$

$z(t) = 3e^{it}+1, \ 0 \le t \le 2\pi$

$S': |w+i| = 5$

$w(t) = 5e^{it} - i, \ 0 \le t \le 2\pi$

$f(3e^{it}+1) = 5e^{it} - i$

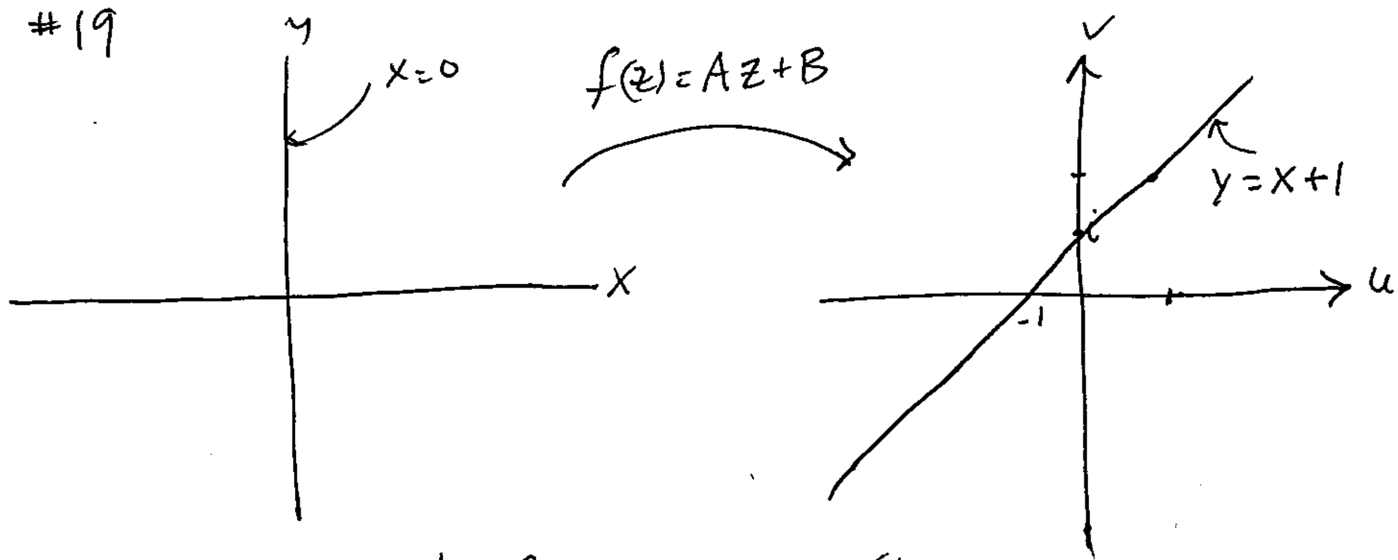
$A(3e^{it}+1) + B = 5e^{it} - i$

$3A = 5 \quad \Rightarrow \quad A = 5/3$

$A + B = -i \qquad B = -i - 5/3$

$$f(z) = \frac{5}{3}z - i - \frac{5}{3}$$

2.3 #19




$z(t)=it, \ -\infty<t<\infty$

$w(t) = t+i(1+t), -\infty\le t\le\infty$
$= i+t(1+i), -\infty\le t\le\infty$

$f(z(t)) = w(t)$

$A(it)+B = i+t(1+i)$

$Ai = 1+i \quad\Rightarrow\quad A = \frac{1+i}{i} = 1-i$

$B = i \qquad\qquad B = i$

$f(z)=(1-i)z+i \leftarrow$ This is a rotation of $-\frac{\pi}{4}$ rad and translation of $+i$.

Rotation 45° followed by translation up one unit

$R(z) = e^{-\frac{\pi i}{4}}z \qquad T(z) = z+i$

$F(z) = T(R(z)) = e^{-\frac{\pi i}{4}}z+i$

$= \frac{1-i}{\sqrt{2}}z+i$

# HW Set 3

8

34. $R(z) = az$, $|a| = 1$ is a rotation, so the image of $z(t) = \rho e^{it}$, $0 \le \rho \le 2$ will also be a disk

$$w(t) = R(z(t)) = \rho a e^{it}, \quad 0 \le t \le 2\pi$$
$$= \rho e^{i\theta} e^{it} \quad \text{where } \theta = \operatorname{Arg}(a)$$
$$= \rho e^{i(t+\theta)} \leftarrow \text{this is also a disk } \rho \le 2$$

Any circle centered at origin is invariant under rotation

(b) $T(z) = z + b$ is a translation by $b$ — $Re(b)$ in horizontal & $Im(b)$ in vertical

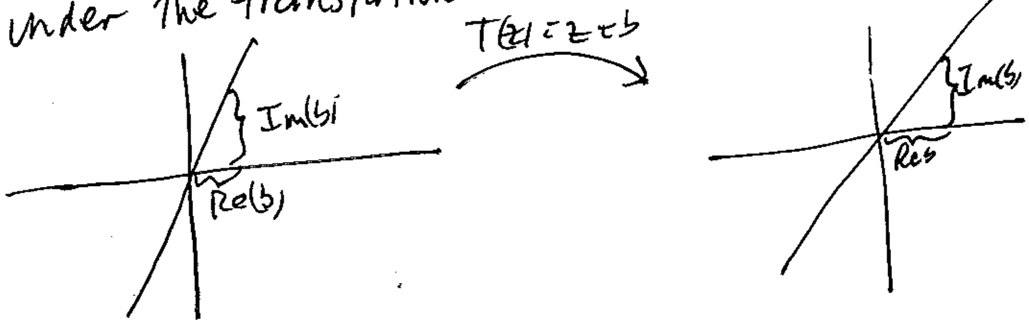
If $Re(b) = 0$ the $T(z)$ is a vertical translation equal to $Im(b)$ units

$\Rightarrow$ Any vertical line $Re(z) = K$ will be invariant under $T$

If $Im(b) = 0$ the $T(z)$ is a horizontal translation equal to $Re(b)$ units

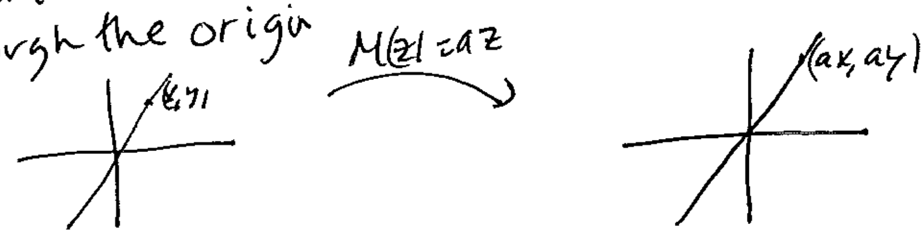
Any horizontal line $Im(z) = K$ will be invariant under $T$

A line through the origin with slope $\frac{Im(b)}{Re(b)}$ will be invariant under the translation $T(z) = z + Re(b) + i\,Im(b)$

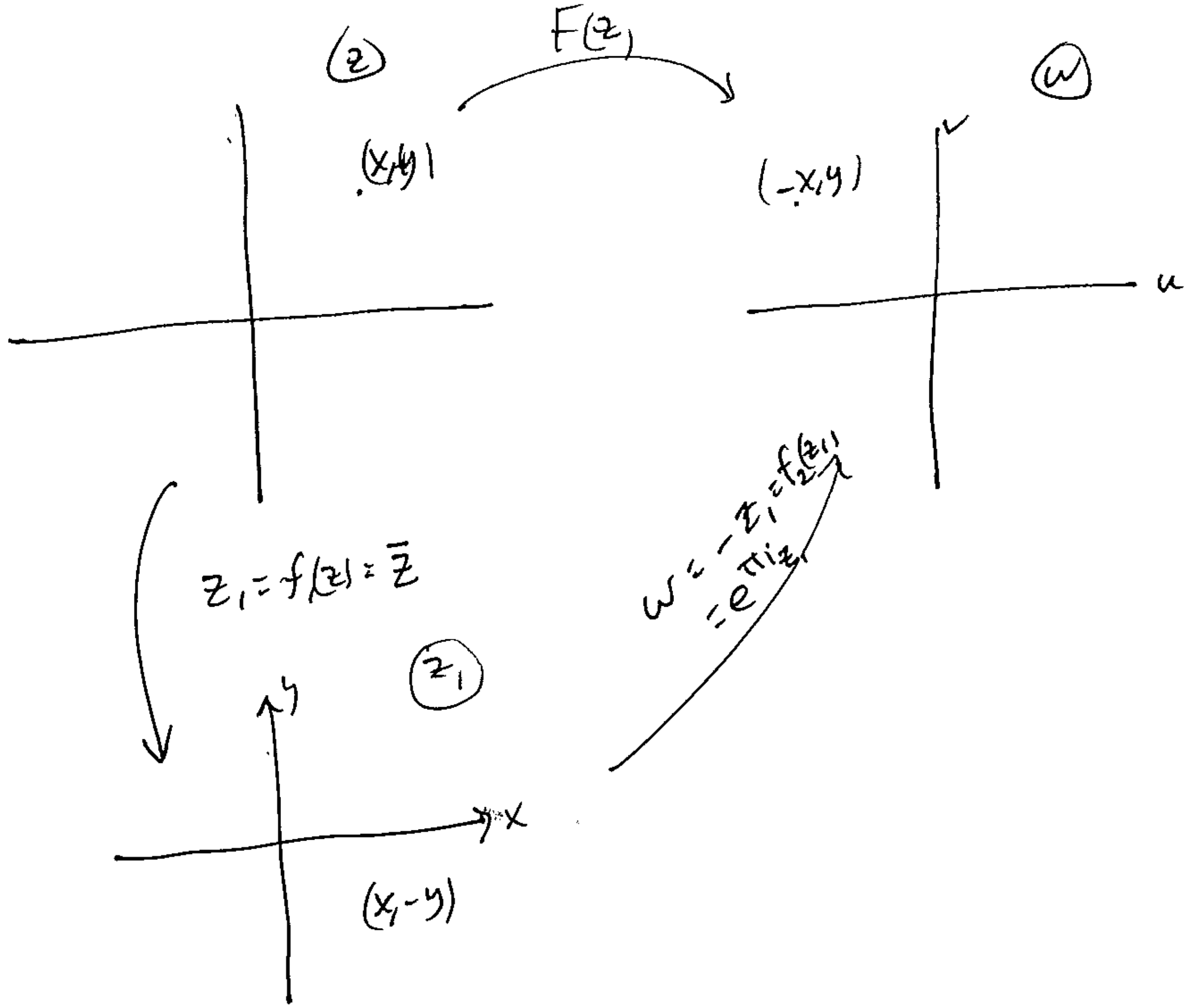



(c) $M(z) = az$

The axes $Re(z) = 0$ and $Im(z) = 0$ are invariant under scaling, as well as ANY line through the origin



29.




$F(z) = f_2(f_1(z)) = -\bar{z} =$

conjugation followed by a rotation by 180° accomplishes reflection about the y-axis