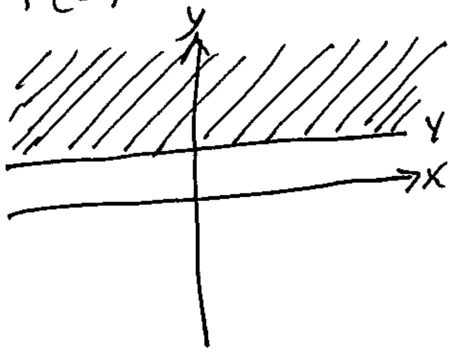
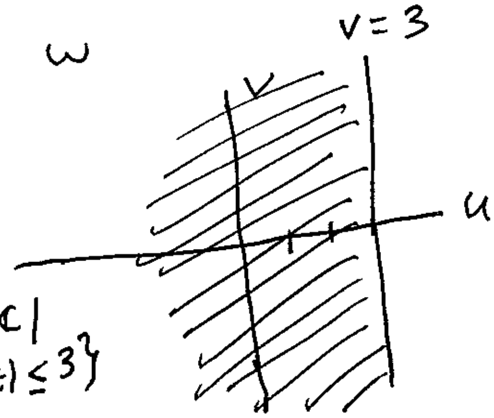
Math 312 HW Set 3

Sec 2.2

#7. $f(z) = iz + 4$



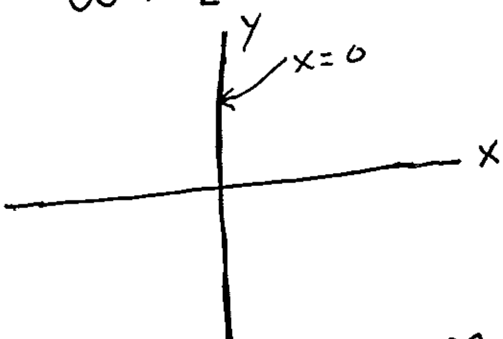


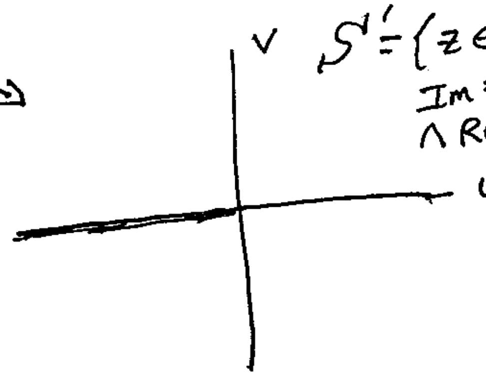
$S' = \{z \in \mathbb{C} \mid Re(z) \le 3\}$

$z(t) = i+t, -\infty < t < \infty$

$$w = f(z(t)) = f(i+t)$$
$$= i(i+t)+4$$
$$= it+3, -\infty < t < \infty$$

#11 $w = z^2 = x^2 - y^2 + 2xyi$



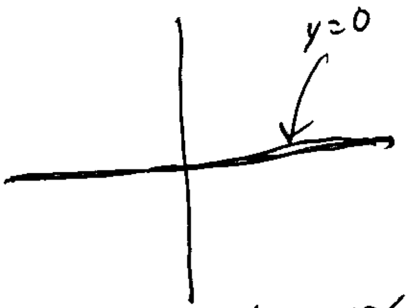


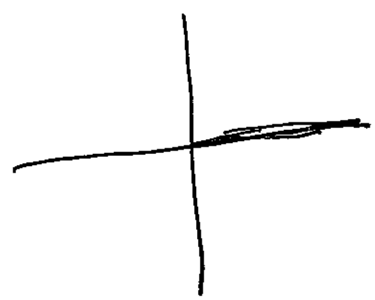
$S' = \{z \in \mathbb{C} \mid Im\, z = 0 \wedge Re\, z < 0\}$

$z(t) = it, -\infty < t < \infty$

$w = (it)^2 = -t^2, -\infty < t < \infty$

#12





$z(t) = t, -\infty < t < \infty$

$w = t^2, -\infty < t < \infty$

$S' = \{z \in \mathbb{C} \mid Im\, z = 0 \wedge Re\, z \ge 0\}$

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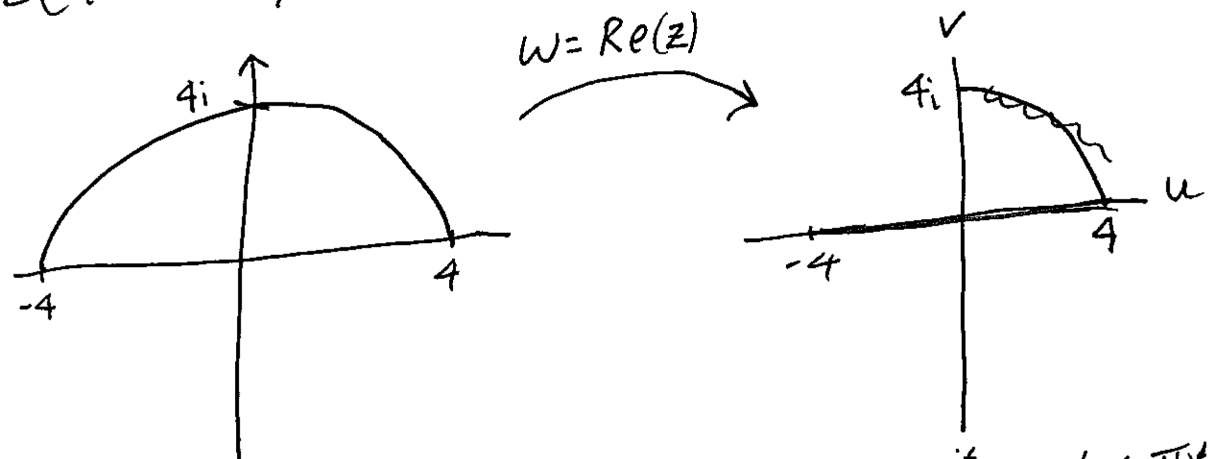
21. $z(t) = 4e^{it},\ 0 \le t \le \pi$

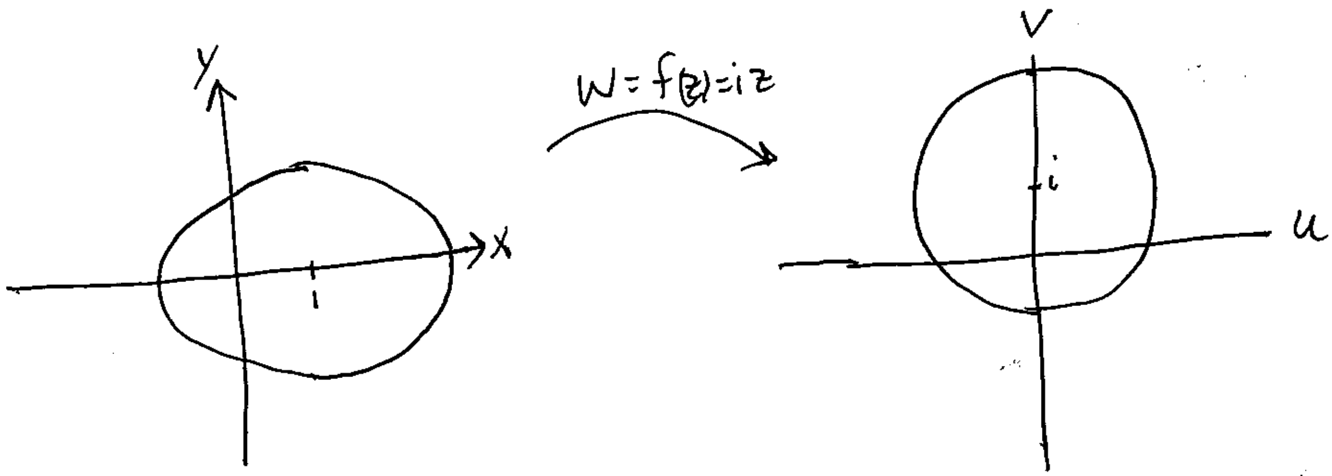


semi-circular arc of radius 4 centered at 0 ~~cse.~~

$w(t) = \cancel{4e^{it}},\ 0 \le t \le \pi$
$4\cos(t)$,

horizontal line between -4 to 4 along y=0 (x-axis)

22.



$z(t) = 2e^{it} + 1,\ 0 \le t \le 2\pi$

$|z-1| = 2$

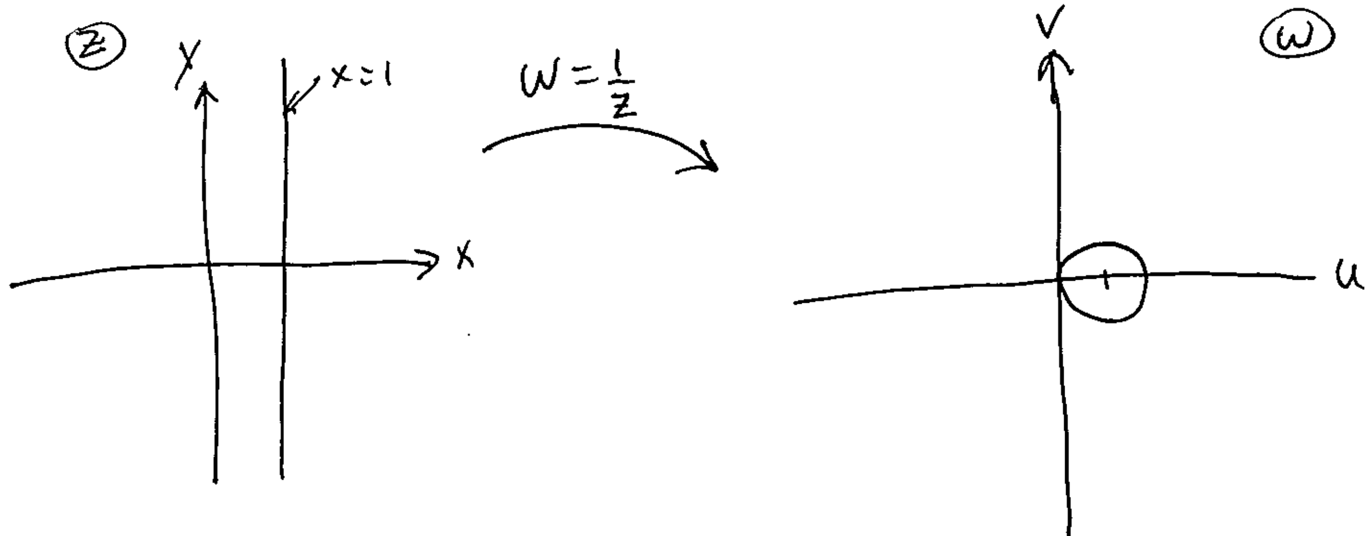
$w = f(2e^{it}+1)$

$w = i + 2ie^{it}$

$|w-i| = 2$

HW Set 3

7.



(a)

$$f(z) = \frac{1}{z} = \frac{1}{1+iy} = \frac{1-iy}{1+y^2} = \frac{1}{1+y^2} + i\left(\frac{-y}{1+y^2}\right) \qquad \left|w - \frac{1}{2}\right| = \frac{1}{2}$$

(b)

$$u = \frac{1}{1+y^2}, \quad v = \frac{-y}{1+y^2}$$

$$u^2 = \frac{1}{(1+y^2)^2} \quad v^2 = \frac{y^2}{(1+y^2)^2}$$

$$u^2+v^2 = \frac{1}{(1+y^2)^2} + \frac{y^2}{(1+y^2)^2} = \frac{1+y^2}{(1+y^2)^2} = \frac{1}{1+y^2}$$

$$u^2+v^2 = u$$

$$u^2 - u + v^2 = 0$$

$$\left(u-\frac{1}{2}\right)^2 + \frac{1}{4} + v^2 = 0$$

$$\left(u-\frac{1}{2}\right)^2 + v^2 = \left(\frac{1}{2}\right)^2$$

(c) This is a circle of radius $\frac{1}{2}$ centered at $\left(\frac{1}{2}, 0\right)$

(d) The point at ∞ in z maps to the origin.

The origin is <u>ALWAYS</u> included in the image of a line under the inversion mapping.

No need to change the image.