

MATH 312 HW Set 3

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Sec 2.1: (3), 8, 14, (20), 36, (27)

Sec 2.2: (7), 11, 12, (23), (27)

Sec 2.3: (9), 18, (19), 34, (29)

Sec 2.4: (23), (25), 31, (47)

Sec 2.5: (4), (16), 22, (25)

2.1 #3 (a) $\log_e(1) + i \text{Arg}(1) = 0.$

(b) $\log_e(4i) + i \text{Arg}(4i) = \ln 4 + i \frac{\pi}{2}$

(c) $\log_e|1+i| + i \text{Arg}(1+i) = \ln \sqrt{2} + i \frac{\pi}{4}$

#8 (a) $r = 2 \quad \theta = \pi$
 $f(z) = r \sin 3\theta + i \cos 2\theta$
 $= 3 \sin 3\pi + i \cos 2\pi$
 $= -3 + i$

(b) $r = \sqrt{2} \quad \theta = \pi/4$
 $f(\sqrt{2} e^{i\pi/4}) = \sqrt{2} \sin \frac{3\pi}{4} + i \cos \frac{\pi}{2}$
 $= \sqrt{2} \cdot \frac{1}{\sqrt{2}} + i \cdot 0 = 1$

(c) $r = 5, \quad \theta = -\frac{\pi}{2}$
 $f(5 e^{-i\pi/2}) = 5 \sin(-\frac{3\pi}{2}) + i \cos(-\pi)$
 $= 5 \cdot (-1) - i$
 $= -5 - i$

#14 $f(z) = z + \frac{1}{z} = x + iy + \frac{1}{x + iy} = x + iy + \frac{x - iy}{x^2 + y^2}$
 $= x + \frac{x}{x^2 + y^2} + i \left(y - \frac{y}{x^2 + y^2} \right)$

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10. $f(z) = z + \frac{1}{z} = r e^{i\theta} + \frac{1}{r e^{i\theta}}$
 $= r e^{i\theta} + \frac{1}{r} e^{-i\theta}$
 $= r \cos \theta + i r \sin \theta + \frac{1}{r} \cos \theta - \frac{1}{r} i \sin \theta$
 $= \left(r + \frac{1}{r}\right) \cos \theta + i \left(r - \frac{1}{r}\right) \sin \theta$

36. $f(z) = \frac{e^{iz} + e^{-iz}}{2}$
 $f(z + 2\pi) = \frac{e^{i(z+2\pi)} + e^{-i(z+2\pi)}}{2}$
 $= \frac{e^{iz} \cdot e^{2\pi i} + e^{-iz} \cdot e^{-2\pi i}}{2}$
 $= \frac{e^{iz} + e^{-iz}}{2} = f(z)$

- 27* (a) $\arg(z)$ is NOT a function (multiple outputs)
 (b) $\text{Arg}(z)$ is a REAL function
 (c) $\cos(\arg(z)) + i \sin(\arg(z)) = e^{iz}$ is a COMPLEX function
 (d) $z^{1/2}$ is NOT a function (2 outputs)
 (e) $|z|$ is a REAL function $|z| = z z^*$
 (f) $\text{Re}(z)$ is a REAL function.

only complex function of complex variable