MATH 312 HW Set 2

Sec 1.4 : 4, 5, 17, 18, 20, 29*
sec 1.5 : 2, 8, 13, 17, 20, 32, 40*
Chap 1 : 8, 15, 21, 30, 32, 45*

1.4

4. $(-125)^{1/3} = |-125|^{1/3} e^{\frac{i(\operatorname{Arg}(-125)+2K\pi)}{3}}, \quad K = 0, 1, 2$

$= 5e^{i\frac{\pi}{3}}, \ 5e^{i\pi}, \ 5e^{\frac{5\pi i}{3}} = 5e^{-\frac{\pi i}{3}}$

5. $(i)^{1/2} = |i|^{1/2} e^{i(\operatorname{Arg} i + 2K\pi)\frac{1}{2}}, \quad K = 0, 1$

$= 1 e^{\frac{i}{2}(\frac{\pi}{2}+2K\pi)}, \quad K = 0, 1$

$= e^{\frac{i\pi}{4}}, \ e^{\frac{5\pi i}{4}}$

17. $z^4 + 1 = 0 \Rightarrow z^4 = -1 \Rightarrow z = (-1)^{1/4}$

$z = |-1|^{1/4} e^{\frac{i}{4}(\operatorname{Arg}(1)+2K\pi)}, \quad K = 0, 1, 2, 3$

$= 1 \cdot e^{\frac{i}{4}(\pi + 2K\pi)}$

$= e^{\frac{i\pi}{4}(2K+1)}, \quad K = 0, 1, 2, 3$

$= e^{\frac{i\pi}{4}}, \ e^{\frac{3\pi i}{4}}, \ e^{\frac{5\pi i}{4}}, \ e^{\frac{7\pi i}{4}}$

18. $z^2 - 8z + 16 = 8i$

$(z-4)^2 = 8i$

$z - 4 = \sqrt{8i} = |8i|^{1/2} e^{\frac{i(\operatorname{Arg}(8i)+2K\pi)}{2}}, \quad K = 0, 1$

$= \sqrt{8}\, e^{i(\frac{\pi}{4}+K\pi)} = \sqrt{8}e^{\frac{i\pi}{4}}, \ \sqrt{8}e^{\frac{5\pi i}{4}}$

$= \sqrt{8}\left(\frac{1+i}{\sqrt{2}}\right), \ \sqrt{8}\left(\frac{-1-i}{\sqrt{2}}\right) = 2\sqrt{2}\left(\frac{1+i}{\sqrt{2}}\right), \ 2\sqrt{2}\left(-\frac{1-i}{\sqrt{2}}\right) = 2+2i, \ -2-2i$

$z - 4 =$ ~~[illegible]~~

$z = \cancel{8 \pm 4i} \ 6 \pm 2i$

$(z-4)^2 = (2+2i)^2 = 2^2 - 4 + 2 \cdot 2 \cdot 2i = 8i$

$z - 4 = \pm(2+2i) \Rightarrow z = 4 \pm (2+2i) = 6+2i, \ 2-2i$

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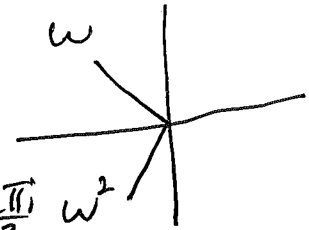
1.4

20. $w = e^{\frac{2\pi i}{3}}$

(a) $w^2 = e^{\frac{4\pi i}{3}} = e^{-\frac{2\pi i}{3}}$

$\overline{w} = w^2$



(b) $1 + w + w^2 = 1 + e^{\frac{2\pi i}{3}} + e^{-\frac{2\pi i}{3}}$

$= 1 + \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$\quad + \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$

$= 1 + 2\cos\frac{2\pi}{3}$

$= 1 + 2\left(-\frac{1}{2}\right)$

$= 1 - 1 = 0$

(c) $w^3 = 1$

$w^3 - 1 = 0$

$(w-1)(w^2+w+1) = 0$

so $w^2 + w + 1 = 0$

29 $\sqrt{1+i} = |1+i|^{1/2} e^{i\frac{(\mathrm{Arg}(1+i) + 2k\pi)}{2}}, \quad k = 0, 1$

$= 2^{1/4} e^{i(\frac{\pi}{8} + k\pi)}, \quad k = 0, 1$

$= 2^{1/4} e^{\frac{i\pi}{8}}, \; e^{\frac{9\pi i}{8}} 2^{1/4}$

$e^{\frac{\pi i}{8}} = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$

$= \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} + i\sqrt{\frac{1-\cos\pi/4}{2}}$

$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} + i\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$

$= \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}} + i\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

$= \frac{1}{2^{1/4}}\left(\sqrt{\frac{\sqrt{2}+1}{2}} + i\sqrt{\frac{\sqrt{2}-1}{2}}\right)$

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$= 2\cos^2\frac{\theta}{2} - 1$

$\sqrt{\frac{\cos\theta + 1}{2}} = \cos\frac{\theta}{2}$

$\sqrt{\frac{1-\cos\theta}{2}} = \sin\frac{\theta}{2}$

$\sqrt{1+i} = 2^{1/4} \cdot \frac{1}{2^{1/4}} \sqrt{\frac{\sqrt{2}+1}{2}} + i\sqrt{\frac{\sqrt{2}-1}{2}},$

$-\sqrt{\frac{\sqrt{2}+1}{2}} - i\sqrt{\frac{\sqrt{2}-1}{2}}$

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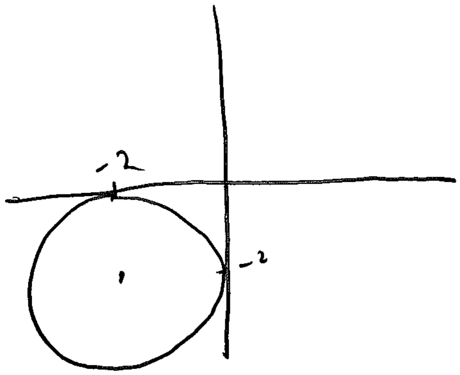
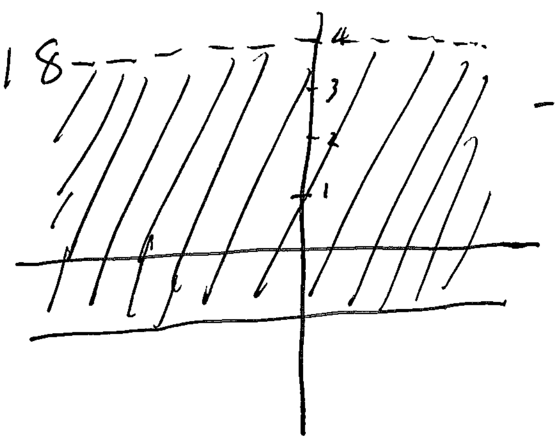
Sec 1.5 : 2, 8, 13, 17, 20, 39, 40*

#2 $|z+2+2i| = 2 \Rightarrow$ Circle of radius 2 centered at $(-2,-2)$



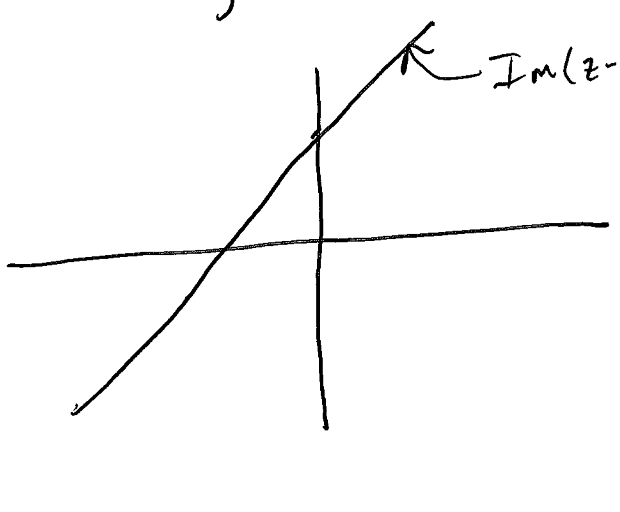
#18



$-1 \leq \operatorname{Im}(z) < 4$

This set is NOT open,
NOT closed
NOT bounded
NOT a domain
IS connected

#8
$$\operatorname{Im}(z-i) = \operatorname{Re}(z+4-3i)$$
$$\operatorname{Im}(x+iy-i) = \operatorname{Re}(x+iy+4-3i)$$
$$y-1 = x+4$$
$$y = x+5$$



$\operatorname{Im}(z-i) = \operatorname{Re}(z+4-3i)$

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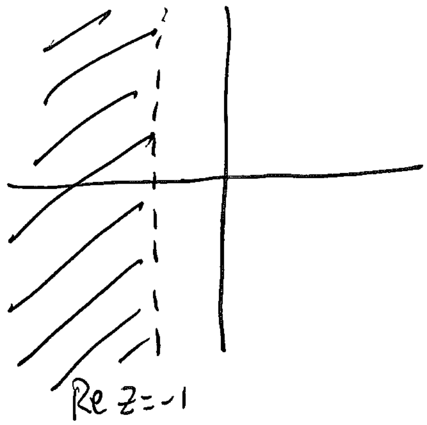
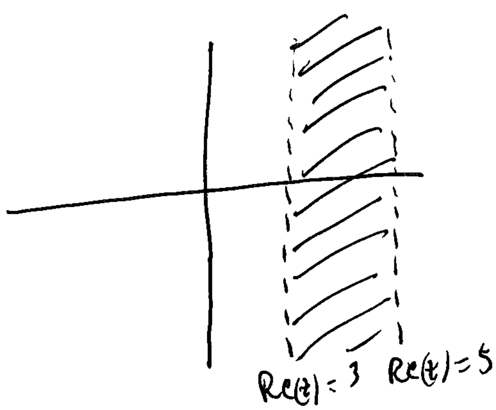
#13 $\text{Re}(z) < -1$



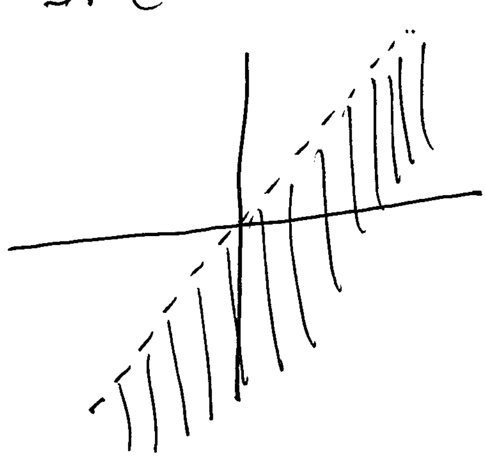
This set is
OPEN
NOT CLOSED
A DOMAIN
NOT BOUNDED
CONNECTED

#17 $2 < \text{Re}(z-1) < 4$



open
connected
domain
not bounded
NOT closed

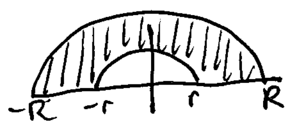
#20 $\text{Im}(z) < \text{Re}(z) \Rightarrow y < x$



open
connected
not bounded
not closed
domain

#39 $\{|z-4i| \geq 4 \cap |z-3| \leq 3\}$

#40* $\{r < |z| < R \cap 0 \leq \text{Arg}(z) \leq \pi\}$



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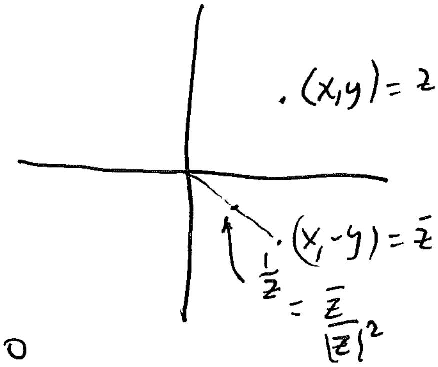
Chap 1: 8, 15, 21, 30, 32, 45*

8. $\arg(\bar{z}) = \arg\left(\frac{1}{z}\right)$

$\arg(x-iy) = \arg\left(\frac{1}{x+iy}\right) = \arg\left(\frac{x-iy}{x^2+y^2}\right)$

TRUE!



15. $|z|<1$ or $|z-3i|<1$
are both open, connected sets, so they are DOMAINS.

21. $\operatorname{Im}(e^{i\theta}) = \operatorname{Im}(\cos\theta + i\sin\theta)$
$= \sin\theta$

30. $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{483} = \left(e^{\frac{i\pi}{3}}\right)^{483}$
$= \left(e^{i\pi}\right)^{161}$
$= (-1)^{161} = -1$ since 161 is odd

32 $i^{127} - 5i^9 - 2i^{-1}$
$= i^3 - 5i - \frac{2}{i}$
$= -i - 5i - 2(-i)$
$= -6i + 2i$
$= -4i$

45* $(1+i)^n = 4096$ $(1+i)^{24} = 4096$

$(1+i)^2 = 2i$
$(1+i)^4 = -4$
$(-4)^{n/4} = 4096 = 2^{12}$
$(-4)^n = 2^{48} = 4^{24} \Rightarrow n = 24$