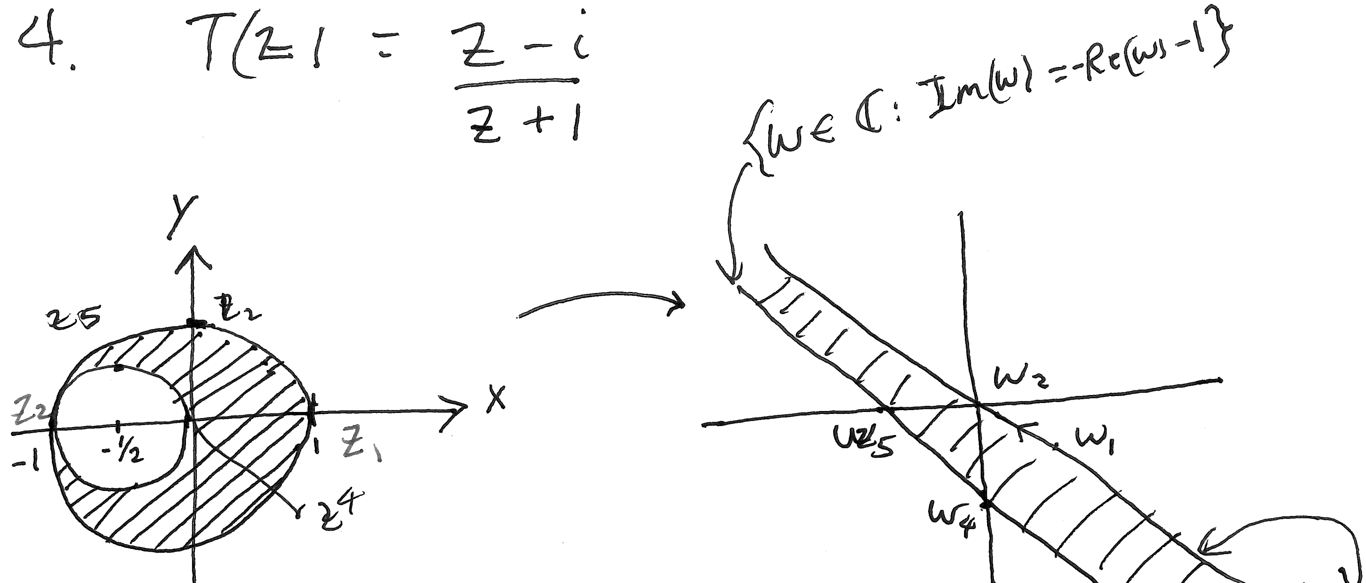
MATH 312 HW11

Sec 7.2: 14, 15, 21 27*

14. $T(z) = \dfrac{z - i}{z + 1}$



$|z| \leq 1 \longrightarrow \{w \in \mathbb{C} : Im(w) \leq -Re(w)\}$

$T(1) = \dfrac{1 - i}{2}$

$T(-1) = \infty$

$T(i) = 0$

$\cancel{T(2i) = \dfrac{i}{1+2i} = \dfrac{i(1-2i)}{1+4} = \dfrac{2}{5} + i\left(\dfrac{1}{5}\right)}$

$\cancel{T(-2i) = \dfrac{-3i}{1-2i} = \dfrac{-3i(1+2i)}{1+4} = \dfrac{6}{5} + i\left(-\dfrac{3}{5}\right)}$

For $|z + \frac{1}{2}| = \frac{1}{2}$ circle becomes a line since pole at $z = -1$ is in pre-image

$T(-1) = \infty$

$T(0) = -i$

$T\left(-\frac{1}{2} + \frac{i}{2}\right) = \dfrac{-\frac{1}{2} + \frac{i}{2} - i}{-\frac{1}{2} + \frac{i}{2} + 1} = \dfrac{-\frac{1}{2} - \frac{i}{2}}{\frac{1}{2} + \frac{i}{2}} = \dfrac{-1 - i}{1 + i} = \cancel{\dfrac{(1-i)(3-i)}{3^2+1^2}} = \cancel{\dfrac{3+-4i}{10}} = \cancel{\dfrac{1-2i}{5}}$

$= -1$

$|z + \frac{1}{2}| = \frac{1}{2} \longrightarrow \{w \in \mathbb{C} : Im\, w = -Re(w) - 1\}$

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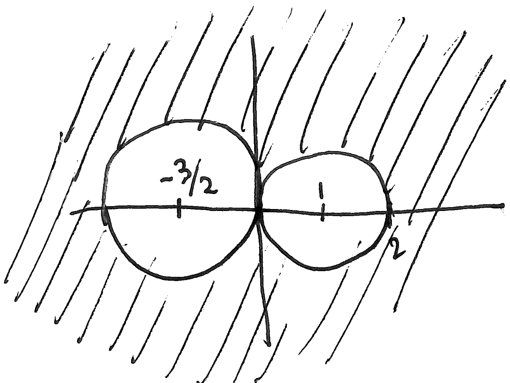
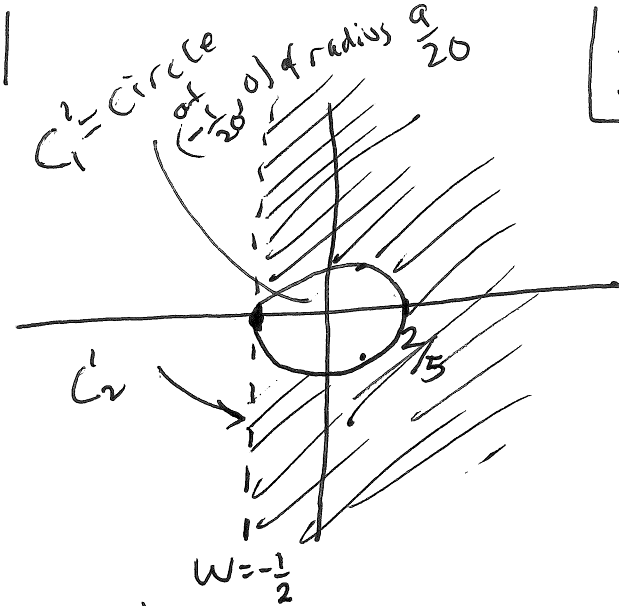
15.





Two circles are

$C_1: \left|z+\frac{3}{2}\right| = 3$

$C_2: |z-1| = 1$

$T(z)=\frac{z+1}{z-2}$

$z=0,\ T(0) = -\frac{1}{2}$

$z=2,\ T(2) = \infty$

$z=1+i,\ T(1+i) = \frac{2+i}{-1+i} = \frac{(2+i)(-1-i)}{1+1} = \frac{-2+1-i-2i}{2} = \frac{-1-3i}{2} = -\frac{1}{2}-\frac{3}{2}i$

C_2 becomes the line $w=-\frac{1}{2}$

For C_1

$T(-3) = \frac{-2}{-5} = \frac{2}{5}$

$T\left(-\frac{3}{2}+\frac{3}{2}i\right) = \frac{-\frac{3}{2}+\frac{3}{2}i+1}{-\frac{3}{2}+\frac{3}{2}i-2} = \frac{-1+3i}{-7+3i} = \frac{(-1+3i)(-7-3i)}{7^2+3^2} = \frac{+7+9-21i+3i}{49+9} = \frac{16-18i}{58} = \frac{8}{29}-\frac{9i}{29}$

$T\left(-\frac{3}{2}-\frac{3}{2}i\right) = \frac{8}{29}+\frac{9i}{29}$

~~center of~~ C_2 gets mapped to a circle centered at $\frac{1}{2}$ way point between $(-\frac{1}{2},0)$ & $(\frac{2}{5},0) \Rightarrow (-\frac{1}{20},0)$ means radius $= \frac{9}{20}$

$\left|w+\frac{1}{20}\right| = \frac{9}{20}$

Check! $\left|\frac{8}{29}-\frac{9i}{29}-\left(-\frac{1}{20}+0i\right)\right| = \frac{9}{20}$

Wolfram ☺

$=$

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21. $z_1 = -1 \qquad w_1 = 0$

$z_2 = 0 \qquad w_2 = 1$

$z_3 = 2 \qquad w_3 = \infty$

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\frac{(w-0)(1-\infty)}{(w-\infty)(1-0)} = \frac{(z+1)(0-2)}{(z-2)(0+1)}$$

$$w = -2\frac{(z+1)}{z-2} = \frac{2z+2}{-2+z}$$

27*

$$\frac{az+b}{cz+d} = \cancel{z}w = f(z)$$

$$\Rightarrow \cancel{cz^2+dz-az-b=0}$$

$$\cancel{z = \frac{-(d-a) \pm \sqrt{(d-a)^2+4\cdot bc}}{2c}}$$

$$az+b = wcz+dw$$

$$az-wcz = dw-b$$

$$z(a-cw) = dw-b$$

$$z = \frac{b-dw}{cw-a} = f^{-1}(w) \qquad cw-a \neq 0$$

because $w \neq \frac{a}{c}$ if z is finite

If $f(z_1) = w \Rightarrow z_1 = \frac{b-dw}{cw-a}$

$f(z_2) = w \Rightarrow z_2 = \frac{b-dw}{cw-a}$

$f(z_1) = f(z_2) \Rightarrow z_1 = z_2$ so f is 1-to-1 function.