HW Set #1

Sec 1.1: 1, 4, 5, 7, 11, 27, 45*
Sec 1.2: 2, 5, 20, 29, 33, 38*
Sec 1.3: 19, 25, 34, 37, 38, 44*

1.1.1 (a) $i^8 = (i^2)^4 = -1^4 = 1$

(b) $i^{11} = i^8 \cdot i^3 = i^3 = -i$

(c) $i^{42} = i^{40} i^2 = 1 \cdot -1 = -1$

(d) $i^{105} = i^{104} \cdot i^1 = 1 \cdot i = i$

1.1.4

$$3(4-i) - 3(5+2i) = 12 - 3i - 15 - 6i$$
$$= -3 - 9i$$

1.1.5

$$i(5+7i) = 5i + 7i^2 = -7 + 5i$$

1.1.7

$$(2-3i)(4+i) = 8 - 12i + 2i - 3i^2$$
$$= 8 - 10i + 3 = 11 - 10i$$

1.1.11

$$\frac{2-4i}{3+5i} = \frac{2-4i}{3+5i} \cdot \frac{3-5i}{3-5i} = \frac{6-12i-10i+20i^2}{3^2+5^2} = \frac{-14-22i}{34}$$

1.1.27

$$\mathrm{Re}\left(\frac{1}{z}\right) = \mathrm{Re}\left(\frac{1}{x+iy}\right) = \mathrm{Re}\left[\frac{x-iy}{x^2+y^2}\right] = \frac{x}{x^2+y^2}$$

1.1.45

when $Z = \bar{Z} \Rightarrow x + iy = x - iy$

$\Rightarrow 2iy = 0 \Rightarrow y = 0$

$\text{Im}\, z = y = 0 \Rightarrow$ so z is purely real

When $Z^2 = (\bar{Z})^2 \Rightarrow (x+iy)^2 = (x-iy)^2$

$x^2 - y^2 + i2xy = x^2 - y^2 - 2xyi$

$4xyi = 0$

so $x = 0$ OR $y = 0$

z is either purely real ($\text{Im}\, z = 0$)

or purely imaginary ($\text{Re}\, z = 0$)

HW Set #1

1.2: 2, 5, 20, 29, 33, 38*

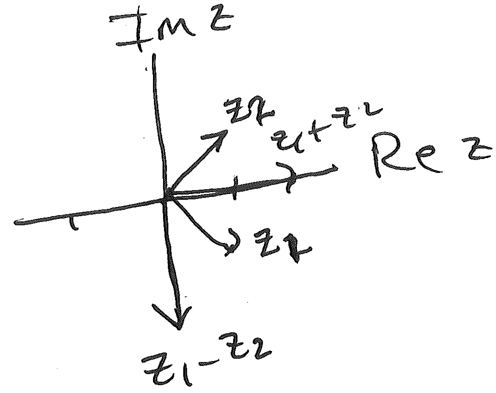
1.2.2

$z_1 = 1 - i$

$z_2 = 1 + i$

$z_1 + z_2 = 2$

$z_1 - z_2 = -2i$



1.2.5

$z_1 = 5 - 2i$

$z_2 = -1 - i$

$z_1 + z_2 = 4 - 3i$

$|z_1 + z_2| = \sqrt{4^2 + 3^2} = 5$

$\cancel{z_3 = 4 \cdot \frac{z_1 + z_2}{|z_1 + z_2|} = \frac{4}{5}(4 - 3i) = \frac{16}{5} - \frac{12i}{5}}$

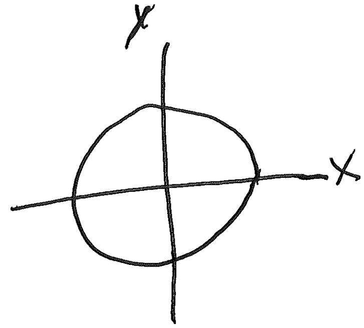
$z_3 = 4(z_1 + z_2) = 16 - 12i$

1.2.20

$\bar{z} = \frac{1}{z} \Rightarrow x - iy = \frac{1}{x + iy} \Rightarrow (x - iy)(x + iy) = 1$

$x^2 + y^2 = 1$

circle of radius 1 centered at 0,0



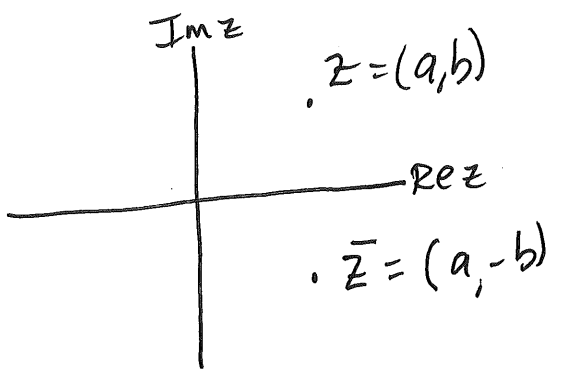
1.2.29

If $|z| \le 1$ Find upperbound of $|3z^2+2z+1|$

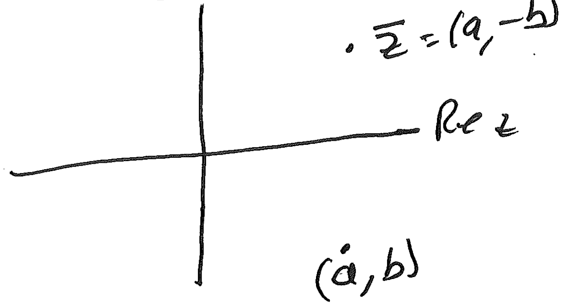
$$
\begin{aligned}
|3z^2+2z+1| &\le |3z^2| + |2z| + |1| \\
&\le 3|z^2| + |2||z| + 1 \\
&\le 3|z|^2 + 2|z| + 1 \\
&\le 3\cdot 1^2 + 2\cdot 1 + 1 \\
&\le 3+2+1 \\
&\le 6
\end{aligned}
$$

1.2.33

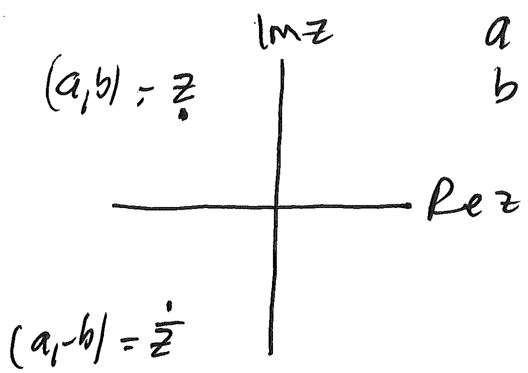
$a>0, b>0$



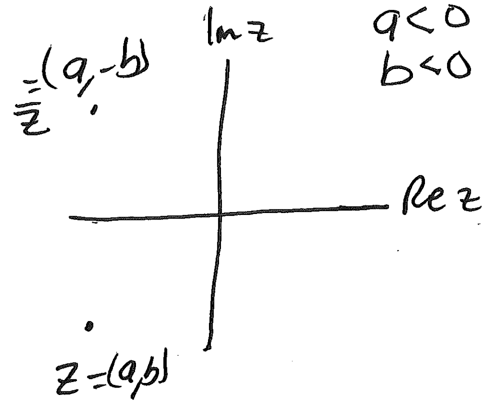
$a>0$
$b<0$



$a<0$
$b>0$



$a<0$
$b<0$



(b) Conjugation geometrically represents REFLECTION across the x-axis, i.e $z = a+ib$ and $\bar{z} = a-ib$

(c) $z = a+ib$ and $z = -a+ib$ represent REFLECTION across the y-axis

HW Set #1

38*

A circle of radius 5 centered at $3-6i$ has equation

$$|z-(3-6i)|=5$$
$$|z-3+6i|=5$$

Since a circle of radius r centered at z_0 has equation

$$|z-z_0|=r$$

In cartesian coordinates this is

$$(x-Re(z_0))^2+(y-Im(z_0))^2=r^2$$

So,

$$(x-3)^2+(y+6)^2=5^2$$

HW Set #1

1.3: 19, 25, 34, 37, 38, 49*

1.3.19 $z_1 = 2(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}) = 2e^{i\pi/8}$

$z_2 = 4(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}) = 4e^{i\frac{3\pi}{8}}$

$z_1 z_2 = (2e^{i\frac{\pi}{8}})\cdot(4e^{i\frac{3\pi}{8}}) = 8e^{\frac{4\pi i}{8}}$

$= 8e^{i\frac{\pi}{2}} = 8i$

$\frac{z_1}{z_2} = \frac{2e^{i\pi/8}}{4e^{i\frac{3\pi}{8}}} = \frac{1}{2}e^{-\frac{2\pi}{8}} = \frac{1}{2}e^{-i\frac{\pi}{4}}$

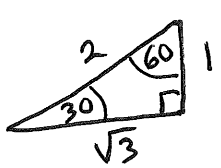
$= \frac{1}{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$

$= \frac{1}{2}\left(+\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} - i\frac{\sqrt{2}}{4}$

1.3.25

$(1+\sqrt{3}i)^9 = \left[\left(1^2+(\sqrt{3})^2\right)^{1/2} e^{i\,Arg(1+i\sqrt{3})}\right]^9$

$= \left[2e^{i\frac{\pi}{3}}\right]^9 = 2^9 e^{3\pi i}$



$= 2\cdot 2^4\cdot 2^4\cdot(-1) = 2\cdot 16\cdot 16\cdot -1$

$= -512$

1.3.34

$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$\cos^3\theta + 3\cos^2 i\sin\theta + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3 =$

$\cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta) = \cos 3\theta + i\sin 3\theta$

$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

1.3.37

(a) $z_1 = -1$ $\quad$ $\text{Arg}\, z_1 = \pi$

$z_2 = 5i$ $\quad$ $\text{Arg}\, z_2 = \frac{\pi}{2}$

$z_1 z_2 = -5i$ $\quad$ $\text{Arg}\, z_1 z_2 = -\frac{\pi}{2}$

$\text{Arg}\, z_1 + \text{Arg}\, z_2 = \pi + \frac{\pi}{2} = \frac{3\pi}{2} \neq \text{Arg}\, z_1 z_2 = -\frac{\pi}{2}$

(b) $\frac{-z_2}{z_1} = \frac{-5i}{-1} = 5i$

$\text{Arg}(-z_2/z_1) = \text{Arg}(5i) = \frac{\pi}{2}$

$-\text{Arg}(z_2) - \text{Arg}(z_1) = -\frac{\pi}{2} - \pi = -\frac{3\pi}{2}$

1.3.38

$z_1 = -1$ $\quad$ $\arg(z_1) = \pi + 2k_1\pi,\ k_1 \in \mathbb{Z}$

$z_2 = 5i$ $\quad$ $\arg(z_2) = \frac{\pi}{2} + 2k_2\pi,\ k_2 \in \mathbb{Z}$

$z_1 z_2 = -5i$ $\quad$ $\arg(z_1 z_2) = -\frac{\pi}{2} + 2k_3\pi,\ k_3 \in \mathbb{Z}$

(a) $\frac{3\pi}{2} + 2k_1\pi + 2k_2\pi = -\frac{\pi}{2} + 2k_3\pi,\ k_3 \in \mathbb{Z}$

$\frac{3\pi}{2} + 2(k_1 + k_2)\pi =$

(b) $\frac{-z_2}{z_1} = 5i$ $\quad$ $\arg\left(-\frac{z_2}{z_1}\right) = \frac{\pi}{2} + 2n\pi,\ n \in \mathbb{Z}$

$-\arg(z_2) - \arg(z_1) = -(\pi + 2k_1\pi) - \left(\frac{\pi}{2} + 2k_2\pi\right)$

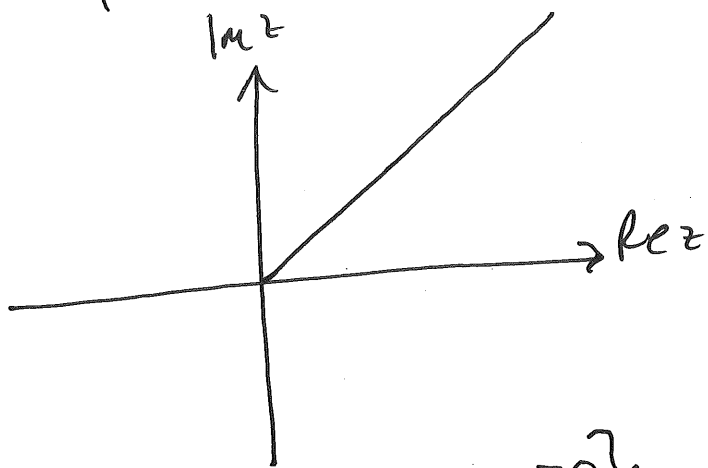
$= -\frac{3\pi}{2} + 2k_1\pi - 2k_2\pi$

$= -\frac{3\pi}{2} + (-2k_1 - 2k_2)\pi$

HW Set #1

44* $\arg z = \pi/4$

This is the set of points in complex plane with $\arg z = \frac{\pi}{4}$



$$\{z \in \mathbb{C} : \operatorname{Re} z = \operatorname{Im} z \wedge \operatorname{Re} z > 0\}$$