Complex Analysis

Math 214 Spring 2014 | Fowler 307 MWF 3:00pm - 3:55pm
 | http://faculty.oxy.edu/ron/math/312/14/

Class 7: Wednesday February 5

TITLE Graphical Interpretation of Complex Linear Functions
CURRENT READING Zill & Shanahan, Section 2.3
HOMEWORK Zill & Shanahan, §2.3 9, 18, 19, 34, **29***.

SUMMARY
We shall focus on the graphical interpretations of the mapping $f(z) = az + b$. We generally can decompose linear complex mappings into 3 dominant characteristics or components. That is, mappings can be described as some combination of **rotation**, **translation** and **magnification**.

Complex Linear Function as Mapping

The complex linear function $f(z) = az + b$ can be written as

$$w = f(z) = az + b = |a| \left(\frac{a}{|a|} z \right) + b$$

which is the composition of the mappings, $f_1(z) = \left(\dfrac{a}{|a|} \right) z$, $f_2(z) = |a|z$ and $f_1(z) = z + b$.

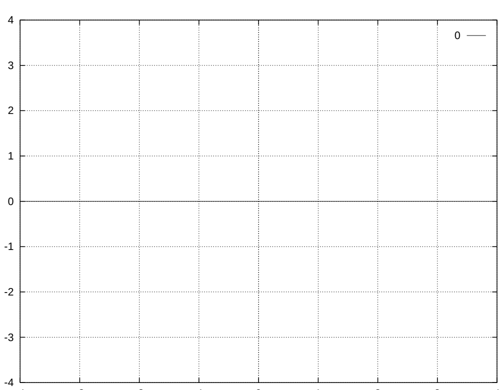
EXAMPLE 1

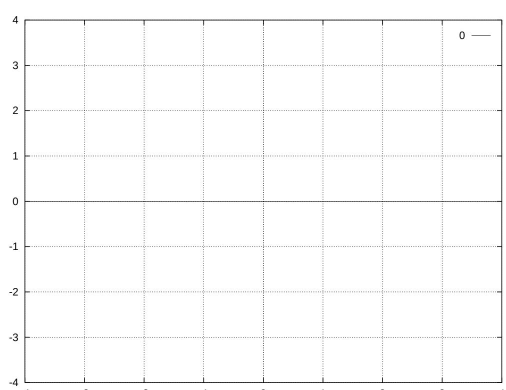
We can show that $f(z) = az + b = f_1(f_2(f_3(z))) = (f_3 \circ (f_2 \circ f_1))(z)$

So, in order, the linear mapping is a ____________________ followed by a ____________________ followed by a ____________________.

Rotation

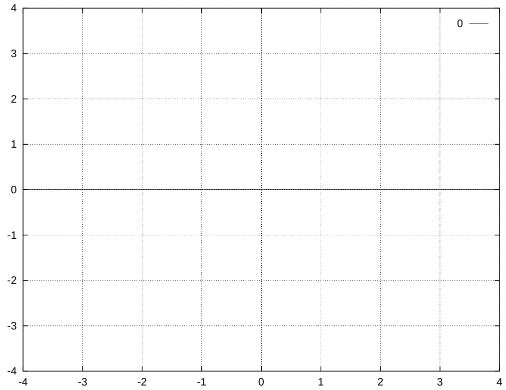
Consider $R(z) = iz$. How does this function represent a rotation mapping?
(Consider its effect on the set of points Im $z = 0$.)

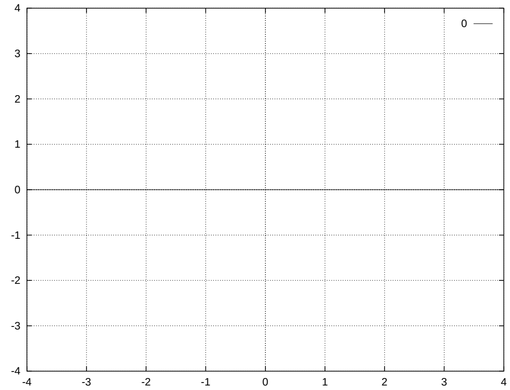




Scaling

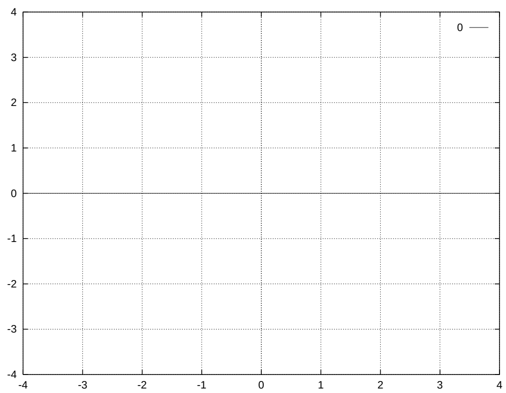
Consider $S(z) = 2z$. How does this function represent a scaling mapping?
(Consider the effect of S on the set of points $|z| = 1$.)

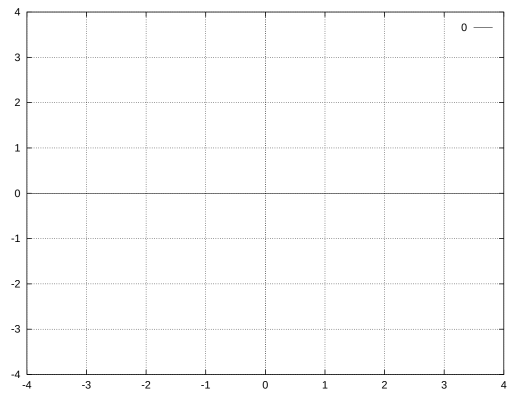




Translation

Consider $T(z) = z - i$. How does this function represent a translation mapping?
(Consider the effect of T on the set of points $|z| = 1$.)





Reflection

Consider $f(z) = \overline{z}$. How does this function represent a reflection mapping?
(Consider the effect of f on the set of points $\mathrm{Im}\ z = 2$.)