Complex Analysis

Math 214 Spring 2004 — Fowler 112 MWF 3:30pm - 4:25pm
 — http://faculty.oxy.edu/ron/math/312/04/

Class 16: **Friday February 27**

SUMMARY Application of Complex Variables: Boundary Value Problems
CURRENT READING Saff & Snider, §3.4
HOMEWORK Saff & Snider, Section 3.4 # 3, 4, 5, 6

Boundary Value Problem

A boundary value problem is very similar to an initial value problem except that in addition to solving a differential equation, the unknown solution must also take specific values at two distinct values.

EXAMPLE

Consider $y'' + 4y = 0, \quad y(0) = 3, y(\pi/4) = -5$
We know that $y(x) = A\sin(2x) + B\cos(2x)$ solves the differential equation so we need to find A and B which satisfy the boundary conditions. What are A and B equal to?

The difficulty with boundary value problems usually occurs as the domain of definition of the problem involves complicated geometries or regions.

We can find solutions of boundary value problems involving Laplace's Equation, by using our knowledge of harmonic functions associated with analytic functions.

Recall that the complex logarithm function $\text{Log}\,(z) = \ln|z| + i\,\text{Arg}\,(z)$ is constant when $|z|$ is constant.
Now, we know that since $\text{Log}\,(z)$ is analytic on the open set not including the negative real axis that its real and complex parts are **harmonic functions**.
Thus if we wanted to solve a boundary value problem involving Laplace's Equation on the domain below...

EXAMPLE

Find a function $\phi(x, y)$ which is harmonic in the annulus centered at the origin with inner radius 1 and outer radius 3. The function takes on the value 7 on the inner boundary and -12 on the outer boundary.

Exercise

Saff & Snider, page 129, # 1. Find the harmonic function $\phi(x, y)$ which satisfies the given boundary value problem.

Exercise

Saff & Snider, page 129, # 3. Find the harmonic function $= \phi(x, y)$ which satisfies the given boundary value problem.