Complex Analysis

Math 214 Spring 2004 Fowler 112 MWF 3:30pm - 4:25pm

 http://faculty.oxy.edu/ron/math/312/04/

Class 12: **Wednesday February 18**

SUMMARY The Complex Exponential
CURRENT READING Saff & Snider, §3.2
HOMEWORK Saff & Snider, Section 3.2 # 1, 4, 5, 9, 12

Now that we know something about analytic functions in general and polynomial functions in particular, we need to expand our repertoire of complex functions.

The Complex Exponential e^z
The complex version of the exponential function is defined like this:
$e^z = e^{x+iy} = e^x(\cos y + i \sin y)$, where $|e^z| = e^x$ and $\arg(e^z) = y + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$
$\arg(e^z) = y + 2k\pi \qquad (k = 0, \pm 1, \pm 2, \ldots)$

Exercise

Show that $f(z) = e^z$ is an *entire function* and that $f'(z) = e^z$
Take some time (3 minutes) to try and prove this. You will have to answer the questions:

1: What is an entire function?

2: How do you show that a function is analytic?

3: Do the real and complex parts of e^z obey the CRE?

More Properties of e^z

- e^z is never zero
- $e^z = 1 \iff z = 2\pi k i$
- $e^{z_1} = e^{z_2} \iff z_1 = z_2 + 2k\pi i, \qquad$ where $k \in Z$
- e^z is a periodic function with period $2\pi i$

A fundamental region of e^z is that set of points in the complex plane which gets mapped to the entire complex plane under the mapping $w = e^z$.
Sketch a *fundamental region* for e^z below

EXAMPLE

Saff & Snider, page 116, #19. Show that the function e^z is one-to-one on any open disk of radius π

Exercise

Howell & Mathews. Show that the image of the first quadrant $\{z : \text{ Re } z > 0 \cap \text{ Im } z > 0\}$ under the mapping $w = e^z$ is the region $\{w : |w| > 1\}$

GroupWork

Saff & Snider, page 117, #25. The behavior of the function $e^{1/z}$ near $z = 0$ is *extremely erratic*. Later (in §5.6) we shall classify this point as an **essential singularity**. Show that you can find values of z, all located in the tiny disk $|z| < .001$ where $e^{1/z}$ takes on the values (a) i, (b) -1, (c) 6.02×10^{23} (Avogadro's number) and (d) 1.6×10^{-19} (charge on a single electron, in Coulombs).