Math 312 Spring 2004

BONUS Quiz 9A

Complex Analysis

Name: ______________________

Date: ______________________

Time Begun: ______________________

Time Ended: ______________________

Friday April 2

Ron Buckmire

Topic : Application of Cauchy Integral Formula

The point of this quiz is to illustrate an application of Cauchy's Integration Formulas to a real integral

Reality Check:

EXPECTED SCORE : ___________**/10** ACTUAL SCORE : ___________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `blackboard.oxy.edu`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday, April 5**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ______________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 312 Spring 2004 **SHOW ALL YOUR WORK** **BONUS Quiz 9A**

We'll show that $I = \int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}$

a. (*2 points*) Show that if $z = e^{i\theta} = \cos\theta + i\sin\theta$ then $\sin\theta = \frac{z - 1/z}{2i}$, $|z| = 1$ and $dz = iz\, d\theta$.

b. (*2 points*) Show that the value of the real integral I is exactly the same as the value of the complex integral $\oint_{|z|=1} \frac{2dz}{4z^2 + 10iz - 4}$

c. (*2 points*) Show that you can write the denominator in (b) as $4(z - z_0)(z - z_1)$ by finding the roots of the quadratic. (HINT: z_0 and z_1 are both have Im $z < 0$ and Re$z = 0$)

d. (*2 points*) Show that the complex integral can be written as $\oint_{|z|=1} \frac{g(z)}{z - z_1}$ where $g(z)$ is completely analytic in and on $|z| = 1$.

e. (*2 points*) Thus the Cauchy Integral Formula tells you that the value of I can be written as a simple formula involving g. What is it?