Complex Analysis

Math 214 Spring 2004 — Fowler 112 MWF 3:30pm - 4:25pm

 — http://faculty.oxy.edu/ron/math/312/04/

Homework Set 11

30 points + 10 journal points + 5 possible bonus points = **40 points**

ASSIGNED: Wed April 21 2004 **DUE: Wed April 28 2004**

1. (*2 pts*) Find a linear transformation which maps the circle $|z+2| = 1$ onto the circle $|w-5| = 3$
2. (*3 pts*) Find a function which maps the upper-half plane to the disk $|2w - i + 1| \leq 1$
3. (*10 pts*) **Saff & Snider, page 392, # 3.** Map $|z - 2| = 1$ under the following mappings (sketch the results). Notice all of these mappings are LFTs of the form $w = \dfrac{az+b}{cz+d}$. For each mapping write down the corresponding values of the parameters a, b, c and d.
   (a) $w = z - 2i$ **(b)** $w = 3iz$ **(c)** $w = 4z + 1 + i$
   (d) $w = \dfrac{z-2}{z-1}$ **(e)** $w = \dfrac{z-4}{z-3}$ **(f)** $w = \dfrac{1}{z}$
4. (*4 pts*) **Saff & Snider, page 392, # 7.** Find the Möbius transformation which maps $0, 1, \infty$ to the following respective points: **(a)** $0, i, \infty$ **(b)** $0, 1, 2$ **(c)** $-i, \infty, 1$ **(d)** $-1, \infty, 1$ After finding $w = T(z)$ sketch the pre-image and image boundaries in the z-plane and $w-$*plane*.
5. *(3 pts)* What is the image of the third quadrant under $w = \dfrac{z+i}{z-i}$?
6. (*3 pts*) **Saff & Snider, page 393, # 11.** Find $M(z)$ which maps the shaded region on the left ($|z| \leq 2 \cap |z-1| \geq 1$) onto the shaded region on the right ($|w-1| \leq 1$).
7. (*5 pts*) **Saff & Snider, page 393, # 12.** Find an LFT which takes the half-plane $x - y < 1$ to unit disk $|w| < 1$ [HINT: Use an LFT which maps both circular boundaries to parallel, horizontal lines. That is, your LFT will map 2 to ∞ and convert the shaded area between circles to a shaded area between lines (a horizontal strip). Then use the exponential mapping to map that strip to the upper-half plane. Then map the upper half-plane to the interior of the unit circle and shift.]

BONUS (5 POINTS):

Saff & Snider, page 393, # 10. Find a conformal map of the semidisk $|z| < 1 \cap$ Im $z > 0$ onto the upper half-plane. [HINT: Use an LFT together with the mapping $w = z^2$. Think about what set you would have to map under $w = z^2$ to produce the entire upper half-plane.]

JOURNAL ENTRY

(*10 points*) Use a separate sheet of paper to discuss your understanding of the mapping process using LFTs. How do you approach problems which involve mappings? What difficulties do you have? Give an example of a problem where you are given a mapping and a pre-image and are asked to find the image *and* also an example where you are asked to find a mapping which transforms a given pre-image to a given image. Discuss how you solve these problems differently. Write atleast three paragraphs. Provide your overall feedback about the homework set. How long did it take you to complete? Which questions were difficult and why?

NOTES

Homework sets are due **one week** from when they are given out. You are strongly encouraged to work collaboratively on the homework and to visit me during office hours to ask questions. Each person must hand in indvidually-written work and indicate with whom they collaborated on the answers. On your first solution page, Indicate the names of the students you worked with.