Complex Analysis

Homework Set 10
30 points + 10 journal points = **40 points**

ASSIGNED: Fri Apr 2 2004   **DUE: Fri April 9 2004**

1. *(4 points)* Find the residue of each function $f(z)$ at $z = 0$
   (a) $f(z) = \dfrac{1}{z^2 + z}$ (b) $f(z) = z\cos\left(\dfrac{1}{z}\right)$ (c) $f(z) = \dfrac{\cot z}{z^4}$ (d) $f(z) = \dfrac{\sinh z}{z^4(1 - z^2)}$

2. *(5 points)* In each case, write the *principal part* of the function at its isolated singular point and determine whether that point is a pole, removable singular point, or an essential singular point:
   (a) $z\exp\left(\dfrac{1}{z}\right)$ (b) $\dfrac{z^2}{1 + z^2}$ (c) $\dfrac{\sin(z)}{z}$ (d) $\dfrac{\cos z}{z}$ (e) $\dfrac{1}{(2 - z)^3}$

3. *(4 points)* Use *Cauchy's Residue Theorem* to evaluate the integral of each of these functions around the circle $|z| = 3$ in the positive sense:
   (a) $f(z) = \dfrac{\exp(-z)}{z^2}$ (b) $f(z) = \dfrac{\exp(-z)}{(z - 1)^2}$
   (c) $f(z) = z^2\exp\left(\dfrac{1}{z}\right)$ (d) $f(z) = \dfrac{z + 1}{z^2 - 2z}$

4. *(4 points)* Use *Cauchy's Second Residue Theorem* to evaluate the integral of each of these functions around the circle $|z| = 2$ in the positive sense:
   (a) $f(z) = \dfrac{z^5}{1 - z^3}$ (b) $f(z) = \dfrac{1}{1 + z^2}$ (c) $f(z) = \dfrac{z^3 e^{1/z}}{1 + z^3}$ (d) $f(z) = \dfrac{(3z + 2)^2}{z(z - 1)(2z + 5)}$

5. *(4 points)* **Saff & Snider, page 314, # 7.** Evaluate $\displaystyle\oint_{|z|=1} e^{1/z}\sin(1/z)\, dz$ (the contour is traversed in the positive sense).

6. *(3 points)* Let C be the circle $|z| = 2$ described in the positive sense, and evaluate the integrals
   (a) $\displaystyle\oint_C \tan(z)\, dz$ (b) $\displaystyle\oint_C \dfrac{dz}{\sinh(2z)}$ (c) $\displaystyle\oint_C \dfrac{\cosh(\pi z)}{(z^2 + 1)z}$

7. *(6 points)* Show that the singular point of each function $f(z)$ is a pole. Determine the order m of the pole and the corresponding residue B.
   (a) $f(z) = \dfrac{1 - \cosh(z)}{z^3}$ (b) $f(z) = \dfrac{1 - \exp(2z)}{z^4}$ (c) $f(z) = \dfrac{\exp(2z)}{(z - 1)^2}$

JOURNAL ENTRY
(10 points) Use a separate sheet of paper to discuss your understanding of residues and singularities. What is a residue and why is it useful to compute it? How do **you** evaluate it? How many different types of singularity do you know? What's the difference between the different types? Comment on the relationship between singularities, residues and contour integrals. You could select a particular problem from this homework set to discuss how these three concepts are linked. Write atleast three paragraphs. Provide your overall feedback about the homework set. How long did it take you to complete? Which questions were difficult and why?

NOTES
Homework sets are due **one week** from when they are given out. You are strongly encouraged to work collaboratively on the homework and to visit me during office hours to ask questions. Each person must hand in indvidually-written work and indicate with whom they collaborated on the answers. On your first solution page, Indicate the names of the students you worked with.