Complex Analysis

Math 214 Spring 2004 — Fowler 112 MWF 3:30pm - 4:25pm

 — http://faculty.oxy.edu/ron/math/312/04/

Homework Set 8

30 points + 10 journal points = **40 points**

ASSIGNED: Fri Mar 12 2004 **DUE: Fri Mar 26 2004**

1. (*4 pts*) **Saff & Snider, p. 160, # 3** Show that the ellipse $x^2/a^2 + y^2/b^2 = 1$ is a smooth curve by producing an admissible parametrization.

2. (*4 pts*) **Saff & Snider, p. 171, # 9** Evaluate $\int_\Gamma (x - 2xyi)dz$over the contour $\Gamma : z = t + it^2, 0 \leq t \leq 1$ where $x =$ Re $z, y =$ Im z.

3. (*4 pts*) **Saff & Snider, p. 171, # 12** True or False: $\oint_{|z|=1} \overline{z}dz = \oint_{|z|=1} \frac{1}{z}dz$.

4. (*3 pts*) $f(z) = (z + 2)/z$ and C is

   (a) the semi-circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq \pi)$

   (b) the semi-circle $z = 2e^{i\theta}$ $(\pi \leq \theta \leq 2\pi)$

   (c) the semi-circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq 2\pi)$

5. (*4 pts*) $f(z) = \pi \exp(\pi\overline{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1 + i$ and i, with the orientation of C being counterclockwise.

6. (*4 pts*) $f(z)$ is defined by the equations

$$f(z) = \begin{cases} 1, & \text{when} \quad y < 0 \\ 4y, & \text{when} \quad y > 0 \end{cases}$$

   and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$.

7. (*3 pts*) Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Without evaluating the integral, show that $\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}$

8. (*4 pts*) Let C_R be the circle $|z| = R (R > 1)$ described in the counterclockwise direction. Show that $\left| \int_C \frac{\text{Log } z}{z^2} \right| \leq 2\pi \frac{\pi + \ln R}{R}$ and then use L'Hospital's Rule to show that as $R \to \infty$ the value of the integral becomes zero.

JOURNAL ENTRY

(*10 points*) Use a separate sheet of paper to discuss the contour integration process in several paragraphs. How do you approach problems which involve contour integrals? Do you understand the parametricization process of the contour and the algorithm for how to find the value of the integral? You can use specific examples drawn from this homework set. Provide your overall feedback about the homework set. How long did it take you to complete? Which questions were difficult and why?

NOTES

Homework sets are due **one week** from when they are given out. You are strongly encouraged to work collaboratively on the homework and to visit me during office hours to ask questions. Each person must hand in indvidually-written work and indicate with whom they collaborated on the answers. On your first solution page, Indicate the names of the students you worked with.