Major Writing Assignment 3

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April 19, 2016

Abstract: Gödel's Incompleteness Theorems

The theorems involve elements of mathematical logic, number theory, and computability. The theorems apply to formal systems, i.e. sets of axioms. The theorems prove that Hilbert's program, which is to find a complete set of axioms for all of mathematics, is not computable. The theorem essentially states $\nexists$ a set of axioms that relates all natural numbers. The incompleteness theorem is not dissimilar from the resulting properties of the undecidable sets in recursive function theory. A set S consisting of natural numbers is r.e. if $\exists$ a Turing machine, or other algorithm, for which the output is the elements of S. The objective is to prove Gödel's first incompleteness theorem and the related lemmas necessary for the proof.

Outline: Gödel's Incompleteness Theorem

Theorem. Gödel's simplified incompleteness theorem: *Suppose that (A, D) is a recursively formal system in which all provable statements are true. Then there is a statement σ that is true but not provable (and consequently $\neg\sigma$ is not provable either)*

Proof. In any recursively axiomatised formal system the set of provable statements P_r is recursively enumerable and $P_r \subseteq T$, where T is the set of all statements that are true in the ordinary arithmetic of natural numbers. T is not r.e. $\Rightarrow$ $\sigma \in T \backslash P_r$. (i.e. σ is true but not provable. Clearly then $\neg\sigma$ is also not provable.

References

1. N.J. Cutland, *Computability: An introduction to recursive function theory*