PDEs in Image Processing

Sarah Greilsamer

April 18th, 2016

Abstract

Before the synthesis of partial differential equations into image processing, images were treated as groups of individual pixels and edited pixel by pixel. In Modern Day image processing, which is a very new area of applied mathematics, images are considered to be continuous functions, and are convoluted with other functions to process them. In two dimensions, the process of denoising, or blurring, happens when the image function is convoluted with the Gaussian function and used to solve the heat equation. In three dimensions, we use the Perona-Malik model to treat our picture. In both cases, solutions to the partial differential equations involve numerical analysis, a tool which will be used to demonstrate how denoising works through real-life example.

Summary

We first introduce an image as a continuous function.

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

2D: We use the convolution of the image function and the Gaussian function,

$$(G_\sigma * u_0)(x) = \int_{\mathbb{R}^2} G_\sigma(x-y) u_0(y) dy$$

to solve the diffusion partial differential equation.

$$\partial_t u(x,t) = \bigtriangledown (C(u,x,t) \bigtriangledown u(x,t))$$

3D: We use the Perona-Malik model.

$$\begin{cases} u_t = \nabla(c(|\nabla u|^2)\nabla u) & \text{in } \Omega \times (0,+\infty) \\ \frac{\partial u}{\partial n} = 0 & \text{in } \partial\Omega \times (0,+\infty) \\ u(x,0) = u_0(x) & \text{in } \Omega \end{cases}$$

References

[1] C.O. Lee, Computational Imaging and Partial Differential Equations, Asia Pacific Mathematics Newsletter (2014)

[2] G. Koepfler, PDE in image processing, retrieved from http://www.math-info.univ-paris5.fr/ gk/ANKARA/pde$_{i}n_{i}m_{p}roc.pdf$

[3] P. Kornprobst, Image Processing using Partial Differential Equations (PDE), retrieved from https://www.lri.fr/ gcharpia/VisionSeminar/slides/2014-04-02-kornprobst-vist.pdf

[4] K. Mikula, Image processing with partial differential equations, Department of Mathematics and Descriptive Geometry

[5] M. Wielgus, Perona-Malik equation and its numerical properties, Uniwersytet Warszawski (2014)

[6] B. Sajadi, Image Processing and Representations, retrieved from http://www.ics.uci.edu/ majumder/PHOTO/ImageProcessingAndRepresentation.pdf

[7]Image Processing: Diffusion Filters, retrieved from http://www1.inf.tu-dresden.de/ ds24/lehre/bvme$_{s}s_{2}013/ip_{0}6_{d}iff.pdf$