Math Comps Part Two Proposal

The roots of -1

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Table of contents

Abstract

Summary of Project Idea

The roots of -1 can be displayed as points on an Argand diagram. The roots will always be evenly spaced along the unit circle. This spacing gives rise to an even more interesting phenomenon—if lines are drawn connecting a single point to all of the $n-1$ other points, the product of the distances of each of these lines will be equal to the degree of z in the equation $z^n - 1 = 0$. It is easy to see this is true by computing the product of the line distances for distinct values of n. However, a proof that generalizes this phenomenon for all positive integer values of n is significantly harder to come by. The objective of my presentation will be to create such a proof.

Equation that needs to be proved

Distance of roots equals order of equation

My comps project will be aimed at proving that for any equation $z^n = -1$:

$$\prod_{k=1}^{n-1} \sqrt{(1 - (cos(\frac{2\pi(k)}{n})))^2 + (-(sin(\frac{2\pi(k)}{n})))^2} = n$$

This means that the product of the distances of the lines going from a single root to each other root is equal to the order of z.